Modelling of fatigue damage progression and life of CFRP laminates

P. PAPANIKOS[1], K. I. TSERPES[2] and SP. PANTELAKIS[2]
[1]*ISTRAM, Institute of Structures and Advanced Materials, Patron-Athinon 57, Patras 26441, Greece,* [2]*Laboratory of Technology and Strength of Materials, Department of Mechanical Engineering and Aeronautics, University of Patras, Patras 26500, Greece*

Received in final form 12 July 2002

ABSTRACT A progressive fatigue damage model has been developed for predicting damage accumulation and life of carbon fibre-reinforced plastics (CFRP) laminates with arbitrary geometry and stacking sequence subjected to constant amplitude cyclic loading. The model comprises the components of stress analysis, fatigue failure analysis and fatigue material property degradation. Stress analysis of the composite laminate was performed by creating a three-dimensional finite element model in the ANSYS FE code. Fatigue failure analysis was performed by using a set of Hashin-type failure criteria and the Ye-delamination criterion. Two types of material property degradations on the basis of element stiffness and strength were applied: a sudden degradation because of sudden failure detected by the fatigue failure criteria and a gradual degradation because of the nature of cyclic loading, which is driven by the increased number of cycles. The gradual degradation of the composite material was modelled by using functions relating the residual stiffness and residual strength of the laminate to the number of cycles. All model components have been programmed in the ANSYS FE code in order to create a user-friendly macro-routine. The model has been applied in two different quasi-isotropic CFRP laminates subjected to tension–compression (T–C) fatigue and the predictions of fatigue life and damage accumulation as a function of the number of cycles were compared with experimental data available in the literature. A very good agreement was obtained.

Keywords CFRP laminates; composites; fatigue; finite element modelling; life prediction; progressive damage modeling.

NOMENCLATURE

A,B,C = experimental fitting parameters
E_{ij} = residual fatigue stiffness
E_{ij}^{d} = degraded elastic moduli
E_{S} = static stiffness
n = number of cycles
N_{f} = number of cycles to failure
L = length of specimen
R = stress ratio, $\sigma_{\min}/\sigma_{\max}$
S_{ij}^{F} = fatigue shear strengths
T = residual fatigue strength
T_{S} = static strength
t = thickness of specimen
$X_{\mathrm{T}}^{\mathrm{F}}$ = fatigue longitudinal tensile strength
$X_{\mathrm{C}}^{\mathrm{F}}$ = fatigue longitudinal compressive strength
$Y_{\mathrm{T}}^{\mathrm{F}}$ = transversal tensile strength

Correspondence: Sp. Pantelakis, Laboratory of Technology and Strength of Materials, Department of Mechanical Engineering & Aeronautics, University of Patras, Patras 26500, Greece. E-mail: pantelak@mech.upatras.gr

Y_C^F = transversal compressive strength
Z_T^F = normal tensile strength
Z_C^F = normal compressive strength
w = laminate width
Δn = increment of number of cycles
ν_{ij} = Poisson's ratios
σ_{ij} = layer stresses
σ_{max} = maximum applied stress
σ_{min} = minimum applied stress

INTRODUCTION

In the last two decades, carbon fibre-reinforced plastics (CFRP) because of their light weight and high specific stiffness and strength are substituting metals in some primary load components in aircraft structures. However, when subjected to high service loads, environmental attack, impact or a combination of any or all of the above, laminated composites develop complex damage mechanisms. As the service load or the time in service increases, damage develops and grows, becoming more severe, and could eventually lead to catastrophic failure. In order to assess durability and damage tolerance requirements of composite structures, fatigue damage must be modelled and methods must be developed to predict life.

For predicting fatigue damage and life of composites, three major approaches have been followed by the researchers until today: the residual stiffness, residual strength and empirical methodologies. Useful reviews about the accomplished work in this field can be found in Ref. [1]. Among three, the residual stiffness approach seems more attractive, because stiffness can be measured non-destructively allowing a more direct experimental support to analytical and numerical models. In each approach, phenomenological, mechanistic, statistical and mixed methods have been utilized. Representative models are the semi-empirical model of Kulkarni *et al.*,[2] the critical element model of Reifsnider[3] and the growth model of Spearing *et al.*[4] All existing models originated from the above three approaches include valuable information but have many crucial limitations, which lead to bad correlation with experimental data and unsuitability for application in general cases. Therefore, the need for development of new models still exists.

In the last few years, new methods based on progressive modelling of damage have been used in order to predict the residual strength of composite laminates with stress concentrations subjected to static loading. The advantages of these methods have increased the potential of use in fatigue loading cases. Nevertheless, the use of progressive damage models in fatigue loading cases is more difficult than in static cases as a result of the complexity of the dynamic loading and are still at the very early stages of development. The only reported attempts are the works of Allen *et al.*[5] and Shokrieh and Lessard.[6]

The model of Allen *et al.*[5] was originally developed in order to simulate the behaviour of microcrack damage in brittle epoxy systems and extended afterwards to toughened polymer systems. The model predicted the growth of intraply cracks for monotonic tensile loadings, and for tension–tension fatigue the associated ply level damage-dependent stress and strain states and the residual strength of laminates with geometric discontinuities. It could also account for the effects of delaminations but only through the use of an empirical relationship that required the supplement of an estimate of the delaminated area. The empirical relationship was necessary to be used because the model was 2-dimensional and could not calculate free edge interlaminar stresses. The damage modes considered were matrix cracking and delamination. The results were in the form of stiffness degradation and postfatigue residual strength. No predictions of fatigue life and damage progression of the laminates could be made with this model.

The progressive fatigue damage model of Shokrieh and Lessard[6] was based on a static model[7] and was applied in bolt-loaded laminates made from the AS4/3501-6 material. The basic difference with the static model was the consideration of gradual degradation of the composite material through the development of a *generalized material property degradation technique*.[6] This technique comprised the components of residual strength, residual stiffness and fatigue life, and required for its application full experimental characterization of the specific material under fatigue loading conditions, which was performed by the authors through a large experimental programme.[8] This characterization is necessary for each different material. The model results were in the form of S–N curves and the comparison with experiments showed a satisfactory agreement. The disadvantage of the model is the requirement of a large number of experiments for each application with a different composite material.

The scope of the present work was to develop a model able to predict fatigue life and damage accumulation as a function of the number of cycles of CFRP laminates subjected to constant amplitude cyclic loading. It was required the model to be parametric with regard to material and cyclic loading conditions, applicable in general cases and to minimize the experimental data needed as input. To this end, a 3-dimensional (3-D) fatigue progressive damage model was developed as an integration of stress analysis, fatigue failure analysis and fatigue material property degradation (sudden and gradual). The gradual degradation of the composite material was modelled by using functions relating the residual stiffness and residual strength of the laminate to the number of cycles. For the derivation of these functions experimental data relating stiffness and strength to the number of cycles at various stress levels are needed. This feature makes the model flexible with regard to application in different materials and fatigue loading conditions. Application of the model was performed in two different CFRP laminates subjected to tension–compression (T–C) fatigue. The predicted S–N curves and fatigue-induced damage accumulation agreed very well with experimental data taken from the open literature.

FATIGUE PROGRESSIVE DAMAGE MODEL

The Fatigue Progressive Damage Model (FPDM) is based on a static Progressive Damage Model (PDM) developed earlier by the authors in Refs [9–11] for predicting the damage accumulation, the load-displacement curve and the residual strength of composite bolted joints subjected to quasi-static tensile loading. The FPDM is an integration of stress analysis, fatigue failure analysis and fatigue material property degradation. In the following sections, a detailed description of the model components and their integration is given.

Fatigue failure analysis

Seven polynomial fatigue failure criteria were used in order to detect seven failure modes. Specifically, for detecting matrix tensile and compressive cracking, fibre compressive failure and fibre-matrix shear-out, a set of 3-D Hashin-type fatigue failure criteria[12] was used, for detecting fibre tensile failure the Maximum Stress criterion[13] was used, and for detecting delamination in tension and compression the Ye-delamination criterion[14] was used. The fatigue failure criteria with respect to failure modes are as follows:

Matrix tensile cracking, for ($\sigma_{yy} > 0$):

$$\left(\frac{\sigma_{yy}}{Y_T^F}\right)^2 + \left(\frac{\sigma_{xy}}{S_{xy}^F}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}^F}\right)^2 \geq 1 \quad (1)$$

Matrix compressive cracking, for ($\sigma_{yy} < 0$):

$$\left(\frac{\sigma_{yy}}{Y_C^F}\right)^2 + \left(\frac{\sigma_{xy}}{S_{xy}^F}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}^F}\right)^2 \geq 1 \quad (2)$$

Fibre tensile failure, for ($\sigma_{xx} > 0$):

$$\left(\frac{\sigma_{xx}}{X_T^F}\right) \geq 1 \quad (3)$$

Fibre compressive failure, for ($\sigma_{xx} < 0$):

$$\left(\frac{\sigma_{xx}}{X_C^F}\right) \geq 1 \quad (4)$$

Fibre-matrix shear-out, for ($\sigma_{xx} < 0$):

$$\left(\frac{\sigma_{xx}}{X_C^F}\right)^2 + \left(\frac{\sigma_{xy}}{S_{xy}^F}\right)^2 + \left(\frac{\sigma_{xz}}{S_{xz}^F}\right)^2 \geq 1 \quad (5)$$

Delamination in tension, for ($\sigma_{zz} > 0$) :

$$\left(\frac{\sigma_{zz}}{Z_T^F}\right)^2 + \left(\frac{\sigma_{xz}}{S_{xz}^F}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}^F}\right)^2 \geq 1 \quad (6)$$

Delamination in compression, for ($\sigma_{zz} < 0$):

$$\left(\frac{\sigma_{zz}}{Z_C^F}\right)^2 + \left(\frac{\sigma_{xz}}{S_{xz}^F}\right)^2 + \left(\frac{\sigma_{yz}}{S_{yz}^F}\right)^2 \geq 1 \quad (7)$$

In the above equations, σ_{ij} are the layer stress components in the ij directions. They refer to a local layer coordinate system, in which the x and y-axes are parallel and transverse to the fibres, respectively, while the z-axis coincides with the normal direction. In the denominators appear the corresponding strengths, in which the superscript F refers to fatigue strengths and the subscripts T, C refer to the tensile and compressive value of the strengths, respectively.

When composite structures are subjected to fatigue loading, the material is loaded by a stress state, which is at the first cycles of loading less than the strength of the material. Therefore, at the first cycles there is no static mode of failure. By increasing the number of cycles, the stiffness and strength of the material degrade as a result of the nature of cyclic loading. The degradation of stiffness leads to stress redistributions and thus, to higher stress states, which in combination with degraded strength values lead to static failures. In order to consider this feature, the degraded strengths were used in the above failure criteria instead of the static strengths. This is the only difference between the static and fatigue failure criteria.

The static Hashin failure criteria[15] have been successfully used in static PDMs in both their original form (e.g. Ref. [16]) and in modified forms (e.g. Refs [6, 9–11])

(Hashin-type failure criteria). In Ref. [11] a parametric study was performed by the authors in order to examine the efficiency of static Hashin-type failure criteria and their effect on predicted load-displacement curves and failure loads of composite bolted joints subjected to quasi-static tensile loading. It was concluded that the consideration of shear stress in fibre tensile Hashin-type failure criterion led to conservative predictions of fibre tensile failure because of high shear stresses developed in the area around the bolt. In order to overcome this problem a new set of failure criteria was proposed by considering the Maximum Stress criterion[13] for fibre tensile failure prediction instead of the Hashin-type criterion. This new set has provided accurate predictions of stiffness and failure load of composite bolted joints[11] and was used in this work also (Eqs (1–5)). Ye-delamination criterion[14] (Eqs (6) and (7)) has provided accurate predictions for free edge delamination onset either in simple composite geometries, such as laminated plates[14,17,18] or in more complex, such as composite bolted joints.[9–11,19] It is assumed that, regardless of the nature of loading, edge delamination is driven by the interlaminar stresses that exist near the free edges of the laminate.

Fatigue material property degradation

In static loading cases, material degradation is applied when a sudden mode of failure is detected by the failure criteria. This type of degradation is called *sudden material property degradation and* is applied by using an appropriate sudden degradation rule, which disables the failed material region from carrying a specific load. In fatigue loading cases, the nature of cycling loading implies an additional material degradation, which is independent of detection of sudden failure and is driven by the increased number of cycles. This type of degradation is called *gradual material property degradation*. Both types of degradations are implemented in this work on element basis and will be discussed in the following sections.

Sudden degradation

When failure is predicted in a ply by the failure criteria of Eqs (1–7), its elastic properties and strengths are degraded by implementing an appropriate *sudden degradation rule*. To date, sudden degradation rules have been applied only in static PDMs. In Ref. [11], the authors have reported work on this topic by examining the effect of sudden material property degradation rules on predicted load-displacement curves and failure loads of bolted composite joints. As a result of this investigation, a new set of sudden degradation rules – which imposed a realistic degradation of the material and led to very good agreement between the predicted and experimental failure loads – was proposed. This set was also used in the FPDM and consists of the following rules (the superscript d refers to the degraded value of the material property):

Matrix tensile cracking:

$$E_{yy}^{\mathrm{d}} = 0.2 * E_{yy} \qquad E_{xy}^{\mathrm{d}} = 0.2 * E_{xy} \qquad E_{yz}^{\mathrm{d}} = 0.2 * E_{yz} \tag{8}$$

Matrix compressive cracking:

$$E_{yy}^{\mathrm{d}} = 0.4 * E_{yy} \qquad E_{xy}^{\mathrm{d}} = 0.4 * E_{xy} \qquad E_{yz}^{\mathrm{d}} = 0.4 * E_{yz} \tag{9}$$

Fibre tensile failure:

$$E_{xx}^{\mathrm{d}} = 0.07 * E_{xx} \tag{10}$$

Fibre compressive failure:

$$E_{xx}^{\mathrm{d}} = 0.14 * E_{xx} \tag{11}$$

Fibre-matrix shear-out:

$$E_{xy}^{\mathrm{d}} = 0 \tag{12}$$

Delamination in tension and compression:

$$E_{zz}^{\mathrm{d}} = E_{yz}^{\mathrm{d}} = E_{xz}^{\mathrm{d}} = 0 \tag{13}$$

Gradual degradation

The gradual degradation of the composite material as a result of the cyclic loading was applied on the basis of element stiffness and strength. The gradual degradation modelling is described in the following two sections.

Modelling gradual degradation of stiffness The residual stiffness of composite laminates subjected to fatigue loading has been studied by many researchers with both theoretical and experimental methods. The residual stiffness has been also used as fatigue damage parameter in order to predict life of the laminates. A large number of the studies (e.g. Refs [20–24]) were concerned with quasi-isotropic CFRP laminates because of their importance in aeronautical applications. These studies have indicated a similar fatigue-induced stiffness degradation for all quasi-isotropic CFRP laminates, which was influenced slightly by the maximum applied stress of the fatigue loading. In particular, stiffness showed an almost linear degradation with respect to the number of cycles.

Figures 1 and 2 show the normalized residual stiffness of quasi-isotropic Fiberdux-HTA/6376 and APC-2 laminates subjected to T–C fatigue for different stress

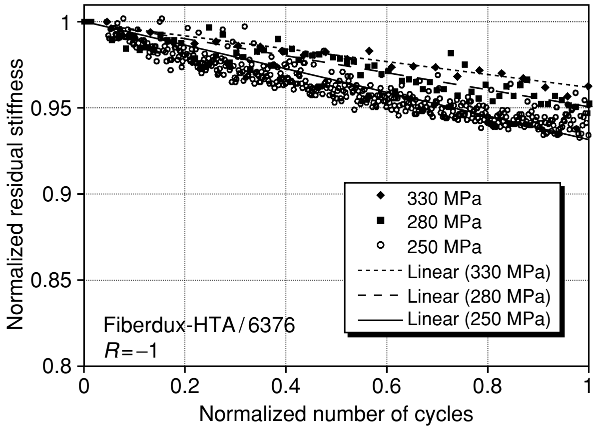

Fig. 1 Normalized residual stiffness of the Fiberdux-HTA/6376 laminate subjected to tension–compression (T–C) fatigue under different stress levels, after Ref. [23].

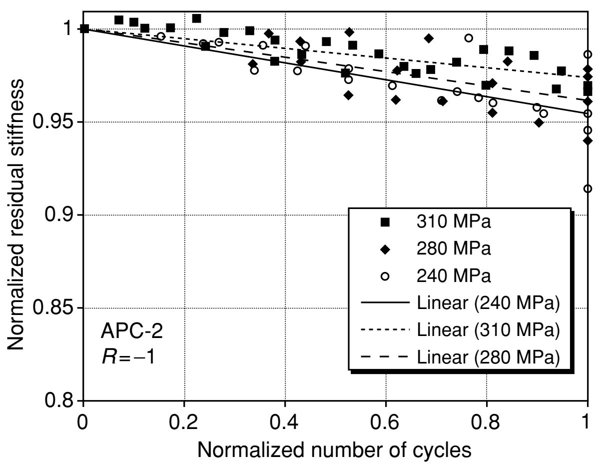

Fig. 2 Normalized residual stiffness of the APC-2 laminate subjected to tension–compression (T–C) fatigue under different stress levels, after Ref. [24].

levels, respectively, obtained experimentally and summarized in Ref. [22]. From these figures, it is clear that the stiffness of the laminates degraded in the manner described above. As may be seen, there is a small influence of the maximum applied stress in the final value of the normalized residual stiffness just before failure. Moving from low to high stress levels the final value is getting bigger. For example, in the case of the Fiberdux-HTA/6376 laminate (Fig. 1) the final value is 0.935 at 250 MPa, 0.94 at 280 MPa and finally 0.963 at 330 MPa.

In the current work, in order to model the gradual degradation of stiffness of the laminates as a function of the number of cycles, linear equations derived from fitting of the data in Figs 1 and 2 for each stress level were used. The general form of the linear equations in terms of normalized residual stiffness and normalized number of cycles is:

$$E(n) = [A(n/N_f) + 1]E_S \tag{14}$$

where E and E_S are the residual and the static stiffnesses, respectively, n the number of cycles, N_f the number of cycles to failure and A an experimental fitting parameter. It is assumed that this degradation applies to all directions of the laminate.

Modelling gradual degradation of strength In the same way to stiffness, the gradual degradation of strength was modelled by fitting the experimental data. Figures 3 and 4 show the experimental normalized residual tensile strength of the two quasi-isotropic CFRP laminates subjected to T—C fatigue loading together with fitting curves for three different stress levels.[22] For the fitting of the normalized residual strength as function of the number of cycles, second-order polynomials were chosen. The general form of the polynomials in terms of normalized residual strength and normalized number of cycles is:

$$T(n) = [B(n/N_f)^2 + C(n/N_f) + 1]T_S \tag{15}$$

where T and T_S are the residual and static strengths, respectively, n the number of cycles, N_f the number of cycles to failure, and B, C experimental fitting parameters. The choice of the polynomials was done because the normalized residual strength degraded in a bigger range than residual stiffness and ended in a smaller value of the static strength at the number of cycles to failure. Equation (15) is used in order to model the degradation of all strength components.

Stress analysis

Consider the laminated fibrous composite plate having arbitrary geometry and stacking sequence shown in Fig. 5. In order to perform stress analysis of the laminate a 3-dimensional (3-D) parametric finite element model

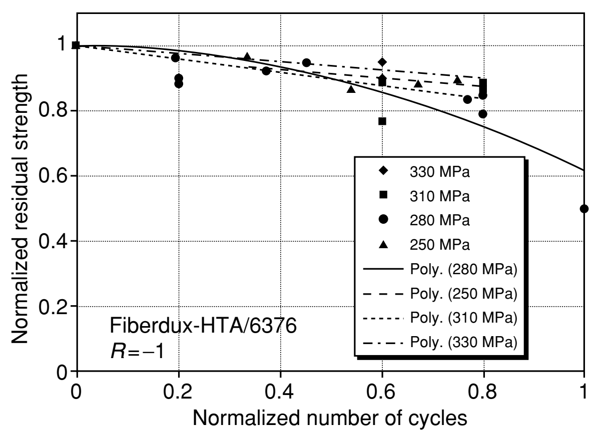

Fig. 3 Normalized residual strength of the Fiberdux-HTA/6376 laminate subjected to tension–compression (T–C) fatigue under different stress levels, after Ref. [23].

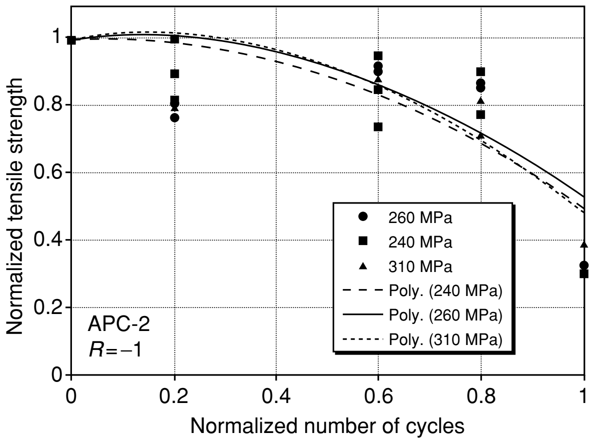

Fig. 4 Normalized residual strength of the APC-2 laminate subjected to tension–compression (T–C) fatigue under different stress levels, after Ref. [24].

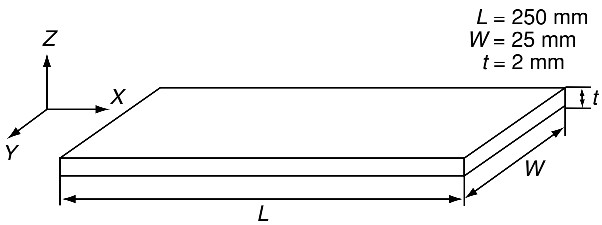

Fig. 5 Schematic draw and dimensions of the laminate.

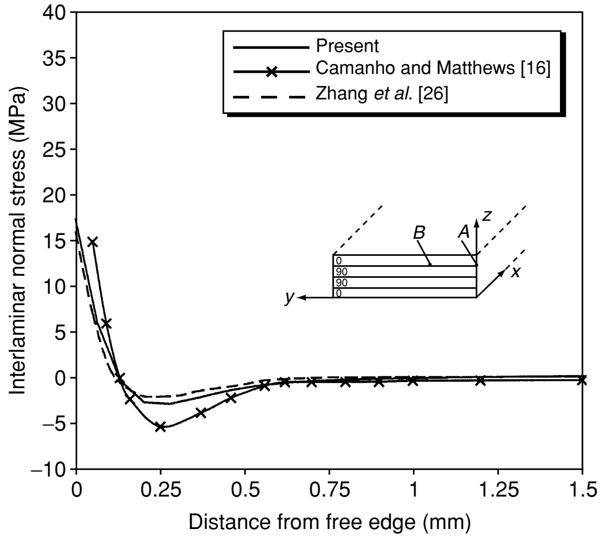

Fig. 6 Comparison of the interlaminar normal stress obtained by the present FE model, the FE model of Camanho and Matthews[16] and analytical solution of Zhang *et al.*[26] Material: $E_{xx} = 137.9$ MPa, $E_{yy} = E_{zz} = 14.5$ MPa, $G_{xy} = G_{yz} = G_{xz} = 5.86$ MPa, $\nu_{xy} = \nu_{yz} = \nu_{xz} = 0.21$, $t_{ply} = 0.125$ mm.

was created by using the ANSYS FE code.[25] The eight-noded 3-D ANSYS SOLID46 layered element with three displacement DOFs per node was used. This element is defined by layer thicknesses, layer material direction angles and orthotropic material properties.

A dense mesh of the laminate (115 through length × 25 through thickness × 8 layers = 23 000 elements) was adopted in order to achieve high accuracy in the calculated stresses, minimize the effect of singularities and capture the high shear and interlaminal shear and normal stresses. The mesh density becomes more important in low stress tension-compression loading cases. In these cases delamination, which initiates at the frees edges and propagates to the centre of the plate is the primary damage mode causing component final failure.[22]

In order to verify the ability of the used layered element to capture with accuracy the through-thickness stresses the problem of a cross-ply $(0°/90°)_S$ laminate loaded in tension was used. The axial dimension was assumed much larger than the other dimensions of the plate and, because of symmetry, only one-fourth of the laminate was modelled. The plate was loaded by a uniform axial strain $\varepsilon_{xx} = 0.01$. The distributions of the through-thickness normal stress σ_{zz} at the 0°/90° interface (AB line in Fig. 6) along the *y*-axis obtained by the present model, the FE model of Camanho and Matthews[16] and the analytical closed form solution of Zhang *et al.*[26] are shown in Fig. 6. It is clear that the present model shows a good agreement to the analytical solution.

Modelling of cyclic loading

In contrast to static loading, the numerical modelling of cyclic loading is a very difficult task. More specific, if it is desired to model the load cycle by cycle the function describing the cycle must be followed exactly. This would demand to choose as many points as possible on the entire function and apply to the FE model the loads that correspond to the selected points. As it can be understood, this is impossible to be performed from the point of view of CPU time and computer storage even for a single cycle and not for the thousands of cycles that are usually applied to real problems.

On the other hand, it is important to consider the effect of the *R* ratio in each problem. For example, in the case of reversed cyclic loading (*R* = *–1*), both the tensile and compressive loads have a significant effect on the developed damage mechanisms and therefore, on the number of cycles to failure.

In the present model, a new methodology has been used in order to apply the cyclic loading in the FE model. This methodology is based on the following assumptions:

- In each cycle, only the maximum and minimum loads are applied based on the assumption that damage initiates only at these loads.

- For the number of cycles equal to the chosen increment only gradual degradation of the material and not sudden is considered.

In order to illustrate the methodology, consider the case of T–C loading ($R = -1$) and the prediction of fibre tensile (fibre breakaway) and fibre compressive (fibre buckling) failure modes through Eqs (3) and (4), respectively. In each cycle, only the σ_{max} tensile load is applied to the laminate, the induced stresses are calculated by the FE model and a check for possible failure modes is performed. For example, the induced tensile $\sigma_x > 0$ stresses are checked with Eq. (3) for possible fibre tensile failure. Then, based on the elastic solution of the laminate, in order to find the stresses that would be induced by the application of the compressive $\sigma_{min} = R\sigma_{max} = -\sigma_{max}$ loading, the previous calculated stresses are multiplied by the R ratio giving in the specific example $R\sigma_x = -\sigma_x < 0$. Finally, the check for possible fibre compressive failure using Eq. (4) is performed. This procedure that incorporates the abovementioned assumptions is repeated for all stress components and for all failure modes and can be followed for any stress ratio R.

Final failure criterion

A very important issue of any progressive damage model is the choice of the final failure criterion. In other words, the decision of the point at which catastrophic failure of the laminate is assumed. In this work, final failure is assumed when the fatigue damage has accumulated in the layers in a degree that disables the laminate from carrying load.

The damage modes that mainly accumulate in fatigued composite laminates and lead to catastrophic failure are varying with respect to loading conditions. At low stress levels, it has been observed experimentally[27,28] that the dominant damage mechanism is delamination, which initiates from the free edges and propagates towards the middle of the laminate. On the other hand, at high stress levels, the dominant damage mechanism is fibre tensile failure.[27,28] Therefore, in the present model, at low stress levels catastrophic failure is assumed when delamination has accumulated critically in the interfaces, and at high stress levels when fibre tensile and compressive (buckling) failures had accumulated critically in the layers. A detailed description of the definition of the final failure criterion at the low and high stress level cases is given in the section entitled 'Damage accumulation'.

Integration of model components

For implementing the FPDM a user-friendly macro-routine, which integrates all model components, was created in the ANSYS FE code. The flowchart of the macro-routine is depicted in Fig. 7 and explained in the following.

1 Development of the 3-D FE model of the composite laminate. Definition of geometry, stacking sequence, initial (static) material properties and loading conditions (maximum applied stress, stress ratio and increment of number of cycles).

2 Performing linear stress analysis in order to calculate and store the 3-D stress field in each element.

3 Performing fatigue failure analysis by applying the failure criteria of Eqs (1–7).

4 Check for ply failures.

4.1 If no failure is predicted, increase the number of applied cycles n by Δn, perform gradual degradation of stiffness and strength of the elements using Eqs (14) and (15) and return to step 2.

4.2 If any mode of failure is detected continue to the next step.

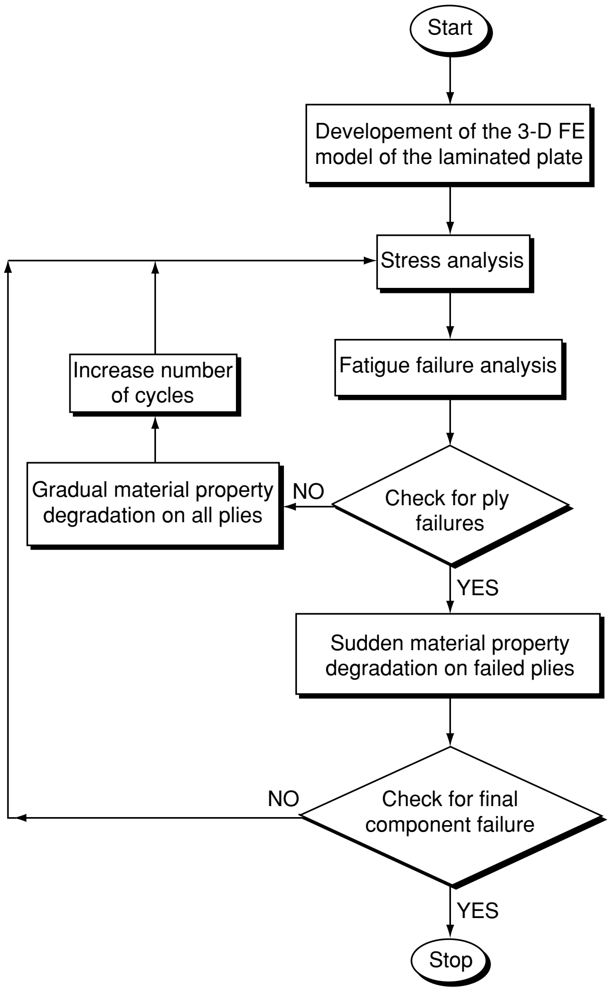

Fig. 7 Flowchart of the fatigue progressive damage model (FPDM).

5 Performing sudden degradation of material properties by applying the rules of Eqs (8–13)
6 Check for final failure of the laminate.
  6.1 Stop, if final failure is reached according to the chosen final failure criterion.
  6.2 If not, return to step 2 and perform again linear stress analysis in order to calculate the redistributed stresses.
7 This procedure is repeated until final failure occurs.

RESULTS AND DISCUSSION

The FPDM was applied to thermosetting Fiberdux-HTA/6376 and thermoplastic APC-2 laminated plates subjected to T–C loading ($R = \sigma_{\min}/\sigma_{\max} = -1$), and the results were compared with the experiments of Refs [23, 24]. The stacking sequences of the HTA/6376 and APC-2 laminates were $(45°/0°/^{-}45°/90_2°/^{-}45°/0°/45°)_S$ and $(45°/0°/^{-}45°/90°)_S$, respectively. The static material properties of the two laminates are shown in Table 1. The dimensions of all specimens were 250 mm × 25 mm × 2 mm, as shown in Fig. 5. The length of the specimens modelled was reduced in both sides by the length of the tabs (50 mm) used in the experiments. The results in form of life prediction and damage accumulation are presented in the following sections.

Life prediction

Figures 8 and 9 show the comparison between the predicted and experimental[23, 24] S–N curves for the two laminates, respectively. The dotted lines in the graphs of these figures are best fits of the experimental data while the solid lines are best fits of the predicted data. For the HTA/6376 laminate, the selected stress levels ranged from 240 to 365 MPa and for the APC-2 from 200 to 310 MPa. For each stress level, the linear and polynomial functions resulted from fitting of the corresponding experimental data were used for modelling the gradual degradation of stiffness and strength, respectively. The procedure described in the sections entitled 'Modelling gradual degradation of stiffness' and 'Modelling gradual degradation of strength' for the stress levels appeared in Figs 1–4 was followed for all the stress levels in the S–N curves.

As may be seen from Figs 8 and 9, a very good agreement, which is within the experimental scatter, has been achieved. In order to give an example, in the case of the HTA/6376 laminate at the load of 330 MPa the mean experimental N_f was 31126 cycles while the predicted was 32400 cycles ($\Delta n = 600$ cycles), and in the case of the APC-2 laminate at the load of 240 MPa the mean experimental N_f was 1.7×10^6 cycles while the predicted was 1.4×10^6 cycles ($\Delta n = 35000$ cycles). Therefore, it can be stated that the FPDM predicts the fatigue life of CFRP composite laminates successfully. The deviation

Table 1 Material properties of the Fiberdux-HTA/6376 and APC-2 laminates

Material property	Fiberdux-HTA/6376	APC-2
E_{xx}	137 GPa	134 GPa
$E_{yy} = E_{zz}$	9.9 GPa	8.9 GPa
$G_{xy} = G_{xz}$	5.2 GPa	5.1 GPa
G_{yz}	3.1 GPa	3.0 GPa
v_{xy}, v_{xz}, v_{yz}	0.3	0.28
X_T	2090 MPa	2130 MPa
$Y_T = Z_T$	75 MPa	80 MPa
$Y_C = Z_C$	168 MPa	1100 MPa
$S_{xy} = S_{xz}$	48 MPa	16
S_{yz}	26 MPa	14

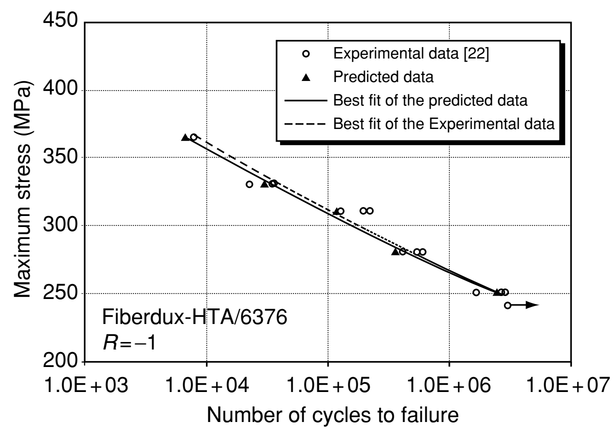

Fig. 8 Predicted and experimental[23] S–N curves of the Fiberdux-HTA/6376 laminate subjected to tension–compression (T–C) fatigue.

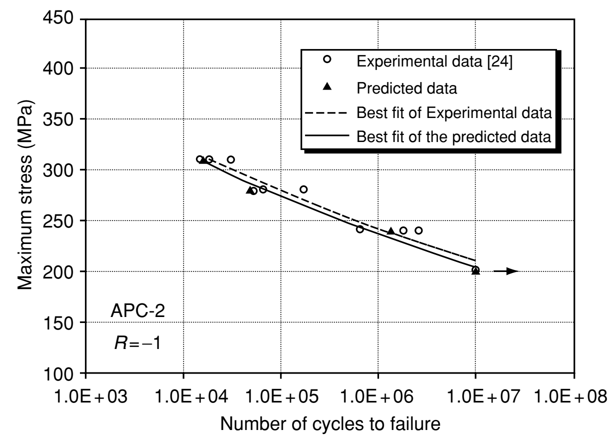

Fig. 9 Predicted and experimental[24] S–N curves of the APC-2 laminates subjected to tension–compression (T–C) fatigue.

observed between the two comparisons of the two laminates is because of the fitting used for the modelling of gradual degradation of stiffness and strength, and because of the experimental scatter concerning the experimental number of cycles to failure.

Damage accumulation

Among the seven damage modes considered, the accumulations of fibre tensile failure and delamination are the most important, because they were used as final failure criteria of the FPDM as explained in the section entitled 'Final failure criterion'. Therefore, this section will concentrate on the discussion of the prediction of accumulation of these two damage types.

In the works of Refs [23, 24], C-scan evaluations have shown that an initial damage pre-existed in the laminated specimens before loading. This damage was delamination observed mainly at the free edges of the specimens, which developed during the manufacturing and cutting processes. Nevertheless, in the present work no initial damage was considered in the laminates modeled, because it was found that that is not changing the predicted damage onset. In fact, the high interlaminar normal stresses at the free edges lead to delamination onset at an early stage of loading.

Figure 10 shows the delamination growth as a function of the number of cycles at the $-45°/0°$ interface of the HTA/6376 laminate for a low stress level case (250 MPa). Delamination initiated at the laminate free edge because of high interlaminar normal stresses caused by compressive loads and propagated towards the middle of the laminate. Similar observations were obtained for the APC-2 laminate also. At the $-45°/0°$ and $0°/-45°$ interfaces of both laminates delamination propagated faster than at the other interfaces. In the APC-2 laminate, extensive delamination was also predicted at the $-45°/90°$ interface. These findings are valid for both laminates for all the low stress level cases and are verified by optical tests of the surfaces of the specimens performed with a microscope after fatigue failure in Refs [23, 24]. At the number of cycles to failure, delamination was predicted in the complete area of the $-45°/0°$ and $0°/-45°$ interfaces and near the free edges at the other interfaces. Matrix cracking was predicted mainly in the 90° layers. Finally, fibre tensile failure and fibre buckling were predicted at the final stage of loading at the 0°, 45° and $-45°$ layers as a result of redistribution of stresses from the matrix to fibres caused by the property degradation of the matrix material, which has failed in matrix cracking and delamination.

In the cases where the laminates were loaded at a high stress level, matrix cracking initiated first at the off-axis layers and especially at the 90° layers at an early stage of loading mainly because of tensile loadings. After matrix cracking, delamination initiated at the free edges of the specimens and close to the areas where matrix cracking was predicted. In the sequence, in half of the laminates' fatigue life, fibre compressive failure (buckling) was predicted at the 0° layers as a result of high compressive on-axis stresses at these layers, which were compared with the small laminate compressive strength in the longitudinal direction (60–70% of the tensile strength). Figure 11 shows the prediction of fibre buckling at the 0° layer of the APC-2 laminate as a function of the number of cycles for the case of high stress level (310 MPa). As may be seen, fibre buckling initiated at the one side of the specimen because of high longitudinal stresses caused by the application of the boundary

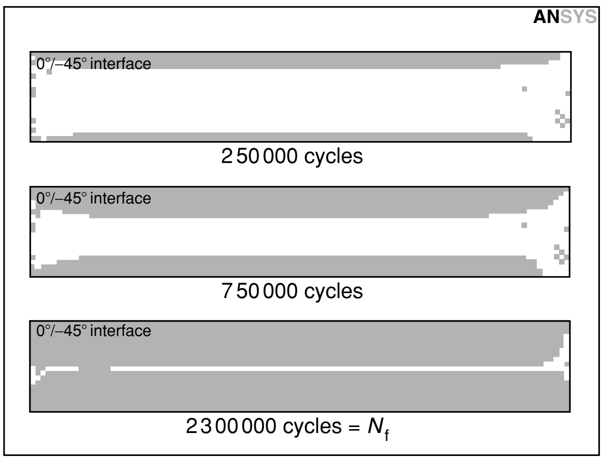

Fig. 10 Prediction of delamination growth as a function of the number of cycles at the $0°/-45°$ interface of the Fiberdux-HTA/6376 laminate subjected to tension–compression (T–C) fatigue (stress level of 250 MPa).

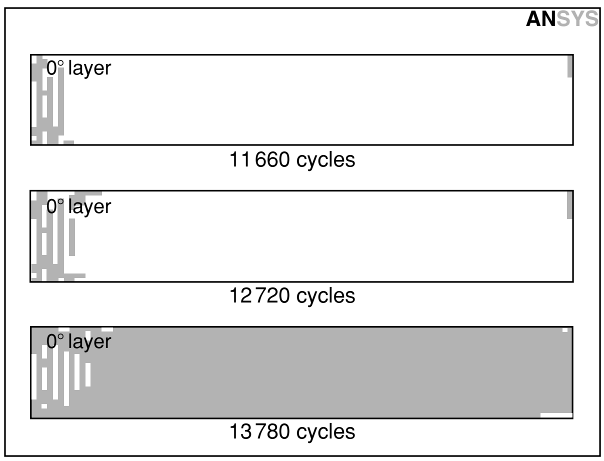

Fig. 11 Prediction of fibre buckling as a function of the number of cycles at the 0° layer of the APC-2 laminate subjected to tension–compression (T–C) fatigue (stress level of 310 MPa).

conditions, and then propagated suddenly from this side to the total area of the layer. The sudden propagation was because of the increase of the stress state in the layer caused by stress redistribution, and because of the gradual degradation of the longitudinal compressive strength of the elements, at the specific cycles. Next, fibre buckling was predicted at the 45° and −45° plies, which are adjacent to the 0° plies caused by redistribution of stresses after sudden material property degradation of the 0° plies. The laminates loaded at high stress levels finally failed as a result of the fibre buckling of the 0°, 45° and −45° plies and as a result of fibre tensile failures (breakaway) at the 0° plies close to the free edges. Delamination was confined at the free edges and did not propagate towards the middle of the specimens as happened at the low stress level cases.

At the two low stress levels at which the laminates did not fail after a large number of cycles in the experiments (3×10^6 at the 240 MPa for the HTA/6376 and 10^7 cycles at the 200 MPa for the APC-2), the model predicted the damage sequence that described previously for the low stress level cases but not in a degree that would satisfy the final failure criterion. In fact, the propagation of damage stopped because of the low applied stress.

It is important to note that the predicted sequence of damage initiation and progression for both low and high stress level cases is verified by experimental observations reported in a series of works.[1,22–24,27,28]

CONCLUDING REMARKS

A 3-D fatigue progressive damage model, developed for predicting damage accumulation and life of CFRP laminates subjected to constant amplitude cyclic loading, was presented. The following concluding remarks can be made:

- The FPDM is parametric with regard to composite material and fatigue loading conditions, and therefore, can be applied in different laminates requiring as input a small number of experiments from which the reduction of residual stiffness and residual strength of the laminates could be modelled as a function of the number of cycles. The requirement of a small number of experiments made the model flexible with regard to the application in different cases.
- The model was applied to thermosetting Fiberdux-HTA/6376 and thermoplastic APC-2 composite laminates subjected to T–C fatigue loading in order to predict the damage accumulation and life. In particular, the predicted S–N curves showed a very good agreement with the experimental, which was better in the case of the HTA/6376 laminates. This deviation was because of the fitting of the experimental data used for the modelling of gradual degradation. The sequence of the onset and growth of the seven different damage modes as functions of the number of cycles was predicted successfully, because it was verified by experimental observations. At the low stress level cases delamination was found to be the dominant failure mechanism, whereas at the high stress level cases fibre tensile failure and fibre buckling.

Acknowledgements

A part of the work presented in this paper was conducted in the frame of the programme 'Bolted Joints in Composite Aircraft Structures' (BOJCAS). The financial support provided by the European Union under Contract G4RD-CT-1999-00036 is gratefully acknowledged.

REFERENCES

1 Reifsnider, K. L. (1990) *Fatigue of Composite Materials*. Elsevier, London.
2 Kulkarni, S. V., McLaughlin, P. V., Pipes, P. B. and Rosen, B. W. (1977) Fatigue of notched fiber composite laminates: an analytical and experimental evaluation. In: *Fourth Conference on Composite Materials: Testing and Design*, ASTM STP 617, pp. 70–92.
3 Reifsnider, K. L. (1986) The critical element model: a modeling philosophy. *Engng. Fract. Mech.* **25**, 739–749.
4 Spearing, S. M., Beaumont, P. W. R. and Kortschot, M. T. (1992) The fatigue damage mechanisms of notched carbon fibre/PEEK laminates. *Composites* **23**(5), 305–311.
5 Allen, D. H., Groves, S. E. and Harris, C. E. (1988) A cumulative damage model for continuous fiber composite laminates with matrix cracking and interply delamination. In: *Eighth Conference on Composite Materials: Testing and Design*, ASTM STP 972 (Edited by J. D. Whitcomb), pp. 57–80.
6 Shokrieh, M. M. and Lessard, L. B. (2001) Progressive fatigue damage modeling of composite materials. Part I: Modelling. *J. Comp. Mater.* **34**, 1056–1080.
7 Lessard. L. B. and Shokrieh, M. M. (1995) Two-dimensional modelling of composite pinned-joint failure. *J. Comp. Mater.* **29**, 671–697.
8 Shokrieh, M. M. and Lessard, L. B. (2001) Progressive fatigue damage modeling of composite materials. Part II: Material characterization and model verification. *J. Comp. Mater.* **34**, 1081–1116.
9 Kermanidis, Th., Labeas, G., Tserpes, K. I. and Pantelakis, Sp. (2000) Finite element modelling of damage accumulation in bolted composite joints under incremental tensile loading. In: *CD-ROM Proceedings of the Third ECCOMAS Congress*, 11–14 September 2000, Barcelona, Spain.
10 Tserpes, K. I., Papanikos, P. and Kermanidis, Th. (2001) A three-dimensional progressive damage model for bolted joints in composite laminates subjected to tensile loading. *Fatigue Fract. Engng Mater. Struct.* **24**, 673–686.
11 Tserpes, K. I., Labeas, G., Papanikos, P. and Kermanidis, Th. (2002) Strength prediction of bolted joints in graphite/epoxy composite laminates, in press.
12 Hashin, Z. (1981) Fatigue failure criteria for unidirectional fiber composites. *J. Appl. Mech.* **48**, 846–852.

13 Tsai, S. W. and Hahn, H. T. (1980) *Introduction to Composite Materials*. Westport Connecticut: Technomic Publishing Co.
14 Ye, L. (1988) Role of matrix resin in delamination onset and growth in composite laminates. *Comp. Sci. Technol.* **33**, 257–277.
15 Hashin, Z. (1980) Failure criteria for unidirectional fibre composites. *J. Appl. Mech.* **47**, 329–334.
16 Camanho, P. P. and Matthews, F. L. (1999) A progressive damage model for mechanically fastened joints in composite laminates. *J. Comp. Mater.* **33**, 906–927.
17 Gamble, K., Pilling, K. and Wilson, A. (1995) An automated finite element analysis of the initiation and growth of damage in carbon-fibre composite materials. *Comp. Struct.* **35**, 265–274.
18 Hu, F. Z., Soutis, C. and Edge, E. C. (1997) Interlaminar stresses in composite laminates with a circular hole. *Comp. Struct.* **37**, 223–232.
19 Camanho, P. P. and Matthews, F. L. (1999) Delamination onset prediction in mechanically fastened joints in composite laminates. *J. Comp. Mater.* **10**, 906–927.
20 Lee, L. J., Fu, K. E. and Yang, J. N. (1996) Prediction of fatigue damage and life for composite laminates under service loading spectra. *Comp. Sci. Technol.* **56**, 635–648.
21 Coats, T. W. and Harris, C. E. (1995) Experimental verification of a progressive damage model for IM7/5260 laminates subjected to tension-tension fatigue. *J. Comp. Mater.* **29**, 280–305.
22 Pantelakis, Sp. and Labeas, G. (2000) Constant and variable amplitude fatigue damage of laminated fibrous composites. In: *Failure Analysis of Industrial Composite Materials* (Edited by E. E. Gdoutos, K. Pilakoutas and C. A. Rodopoulos), pp. 247–295. McGraw-Hill, New York.
23 Pantelakis, Sp.G., Kyriakakis, Em.Ch and Papanikos, P. (2001) Non-destructive fatigue damage characterization of laminated thermosetting fibrous composites. *Fatigue Fract. Engng Mater. Struct.* **24**, 651–662.
24 Pantelakis, Sp.G. and Kyriakakis, Em.Ch (1999) Fatigue damage of APC-2 composite assessed from material degradation and non-destructive evaluation data. *Theor. Appl. Fract. Mech.* **32**, 37–46.
25 *ANSYS User's Manual*, Version 5.7. Swanson Analysis Systems, Inc (2001).
26 Zhang, C., Lessard, L. B. and Nemes, J. A. (1997) A closed-form solution for stresses at curved free edges in composite laminates: a variational approach. *Comp. Sci. Technol.* **57**, 1341–1354.
27 Pantelakis, S. G., Philippidis, T. P. and Kermanidis, T. B. (1995) Damage accumulation in thermoplastic laminates subjected to reversed cyclic loading 1995. In: *Proceedings of the Fourth International Sumposium on High Technology Composites in Modern Applications, Corfu, Greece, 18–22 September 1995* (Edited by S. A. Paipetis and A. G. Youtsos), pp. 156–164. University of Patras Publications, Patras, Greece.
28 Rotem, A. and Nelson, H. G. (1989) Failure of laminated composite under tension–compression fatigue loading. *Comp. Sci. Technol.* **36**, 45–62.