



Specific Targeted Research Project  
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**Thematic Priority 4 - Aeronautics and Space**



# **Post-buckling Analysis of Composite Panels - Towards Fast Tools with Implicit FEM Methods**

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# Introduction



SMR participates in the current EU Framework 6 research project COCOMAT. One of the goals of COCOMAT is to find and verify computational methods for simulating - **as accurately and fast as possible** - the buckling and post-buckling behaviour of composite shell structures.

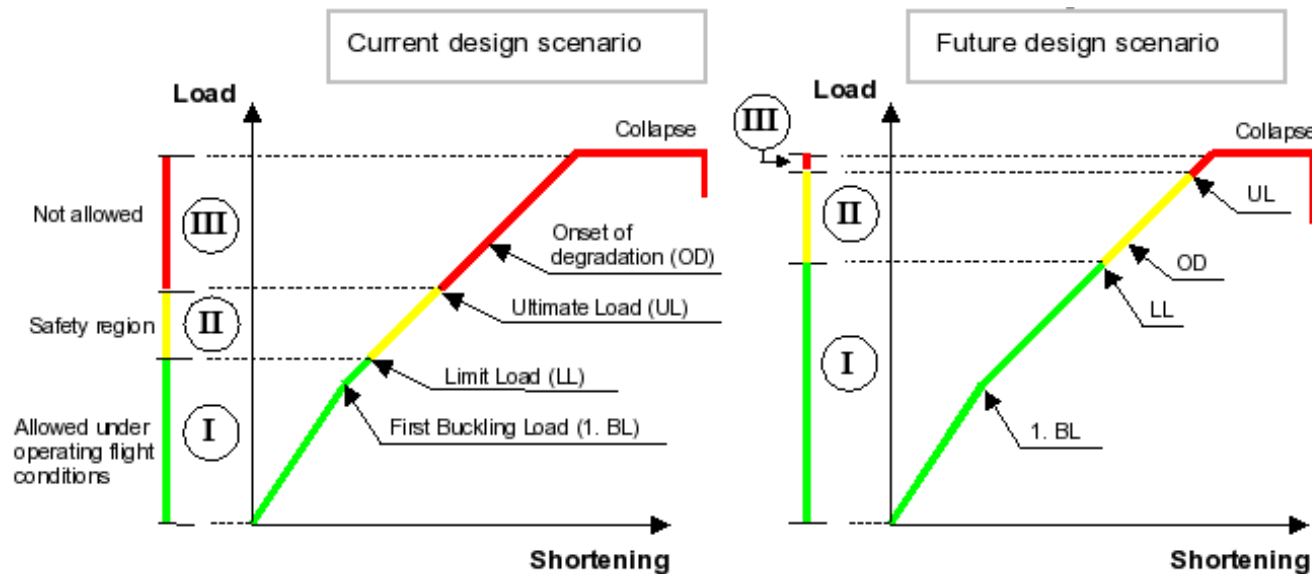
SMR's recent contribution to this specific application of the finite element method has resulted in an increase of the range and efficiency of the methods available in the B2000 Finite Element code. The improvements concern a new composite shell element, a rewritten non-linear static procedure and a non-linear transient solver that together give the user considerably more freedom in conducting panel buckling simulations.



# Post-buckling of composite shell structures

COCOMAT aims at improving design scenarios for string stiffened composite panels:

- Increasing the limit load,
- Postponing onset of damage to higher loads,
- Optimizing exploitation of materials.



# Post-buckling of composite shell structures

Analysis tools available today can be classified by their computational efficiency as follows:

	<b>Speed</b>	<b>Comment</b>
<b>Very slow tools</b>	Days	Explicit codes. Can be improved with parallel solvers.
<b>Slow tools</b>	Hours	Implicit codes. Can to some extent be improved with parallel solvers.
<b>Fast tools</b>	Minutes	Improved implicit codes, reduced basis technique, semi-analytical methods. Can to some extent be improved with parallel solvers.
<b>Very fast tools</b>	Seconds	Empirical methods?



# Post-buckling of composite shell structures

One of the aims of COCOMAT is to improve implicit FEM 'slow' analysis tools by

- Performing more accurate elastic FEM simulations in less time,
- Simulating damage in materials with new degradation models, aiming at getting accurate results with reasonable computational effort.
- Combining implicit FEM 'slow' analysis tools with 'fast' tools based on semi-analytical approaches or on reduced basis techniques in order to obtain fast analysis response for the optimization of panel designs:
  - Semi-analytical analysis procedure for optimizing designs.
  - Full shell analysis for verification.Both with the same input definition.



# SMR's contribution towards a fast tool

Improvement of an implicit code without loss of accuracy by using state of the art numerical and programming methods:

- Composite shell element with **pre-integration through the thickness** results in a faster element.
- **Super-nodal sparse linear solver using optimized BLAS3 routines** results in a faster implicit FEM solver.
- **C++ template meta-programming** techniques result in a faster FEM solver implementation.
- A non-linear implicit iterative solver with well-tuned **variable step size strategy** reduces the number of necessary iterations.



# The B2000 FEM Environment

The B2000 environment is used by SMR for the development of a new COCOMAT improved slow tool.

- B2000 contains an extensive finite element library: Solid mechanics, heat flow, acoustics, also in combination.
- B2000 offers a set of FEM solvers featuring:
  - Implicit linear and non-linear static or transient analysis, eigenvalue analysis, linearised pre-buckling analysis,
  - Non-linear and time dependent boundary conditions,
  - Real and/or complex degrees of freedom leading to symmetric or non-symmetric second variation.
- B2000 is extensible by reusable modules using C++ object oriented programming methods: Fast implementation of new elements, material models, boundary conditions, and FEM solvers.



# The B2000 new element Q4.MITC\_E

A new fast and accurate non-linear shell element for composite structures has been implemented, featuring

- Pre-integration of the material equations through the thickness direction without loss of accuracy.
- Automatic selection of 5 or 6 degrees of freedom per node:
  - 6 degrees of freedom at shell intersections: More accurate.
  - 5 degrees of freedom elsewhere: No rank deficiency (no need for 'artificial drill stiffness').
- State of the art methods to reduce the locking problem:
  - *Assumed Natural Strain* for out-of-plane strain.
  - *Enhanced Assumed Strain* for in-plane strain.
- Use of finite rotations to compute directors from rotational degrees of freedom.



# The B2000 new element Q4.MITC\_E

The new Q4.MITC\_E element resulted in substantial reduction of computational cost:

- Comparison of the new element with the Q4.BA6 element, both based on Reissner-Mindlin shell theory. The time corresponds to the computation of the first and second variation of all the elements of the PSC5 panel model on an AMD Athlon 3500+:

	Q4.BA	Q4.MITC	Speedup
First variation	2.07s	0.035s	59
Second variation	9.77s	0.23s	42

- Conclusion: Speedup is such that parallelization at element level does not enhance processing time significantly for a typical panel FEM mesh (~5000 elements).



# The B2000 implicit static solver

The B2000 static non-linear solver makes use of a predictor-corrector method and a wide range of increment control:

- Load and displacement control
- State control
- Hyperplane control (Riks), local hyper-elliptic control (Crisfield).

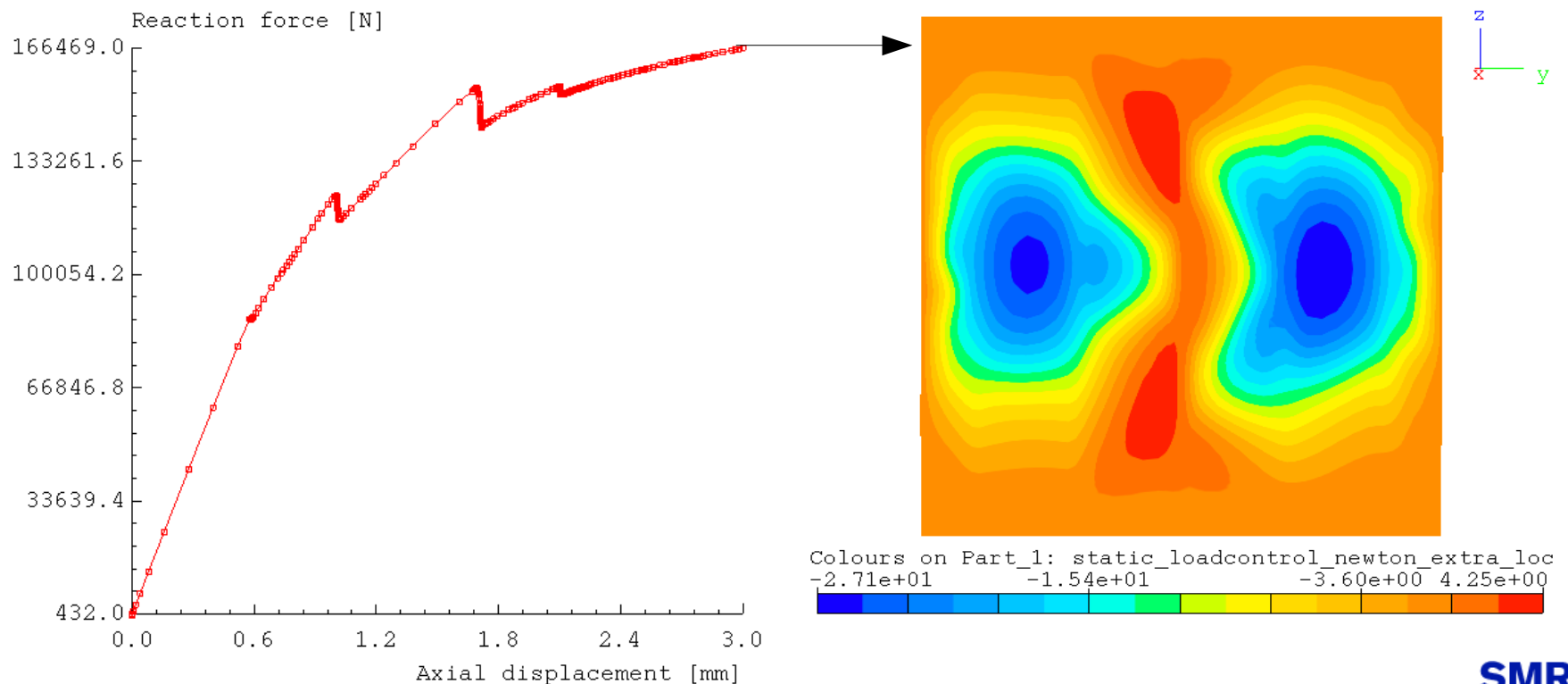
Standard predictor-corrector methods like the continuation methods applied to the PSC5 model exhibit difficulties in passing the multiples bifurcation points encountered along the equilibrium path. To overcome these difficulties, an artificial damping procedure was added to the B2000 static non-linear solver. Combined with a displacement control of the increment, this method delivers satisfactory results with the PSC5 model.



# The B2000 implicit static solver

Post-buckling analysis results with the PSC5 benchmark using the Q4.MITC\_E element:

- First local buckling at 0.59 mm and 87 kN
- First global buckling at 1.01 mm and 123 kN



# The B2000 implicit transient solver

The B2000 transient solver features

- A linear multi-step (BDF - backward difference) time integration method which works well for stiff ordinary differential equations.
- A variable time step:
  - The Nordsieck transformation method is applied to the BDF method to obtain a variable time-step time-integration method.
  - The time-step is controlled by a time-integration error estimation that makes use of Milne's device method: The error estimate is obtained by comparing the result of the implicit integration to the one obtained with an explicit Nordsieck linear multi-step method of the same order (Adams Bashforth).

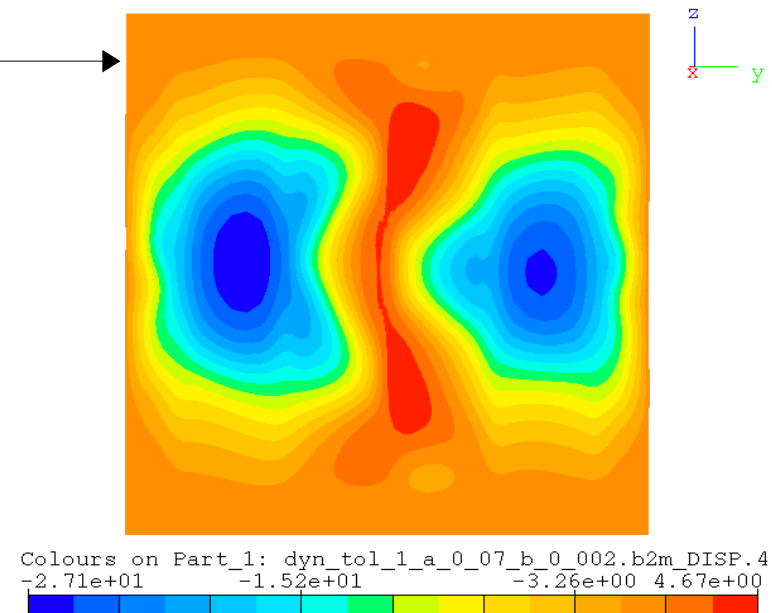
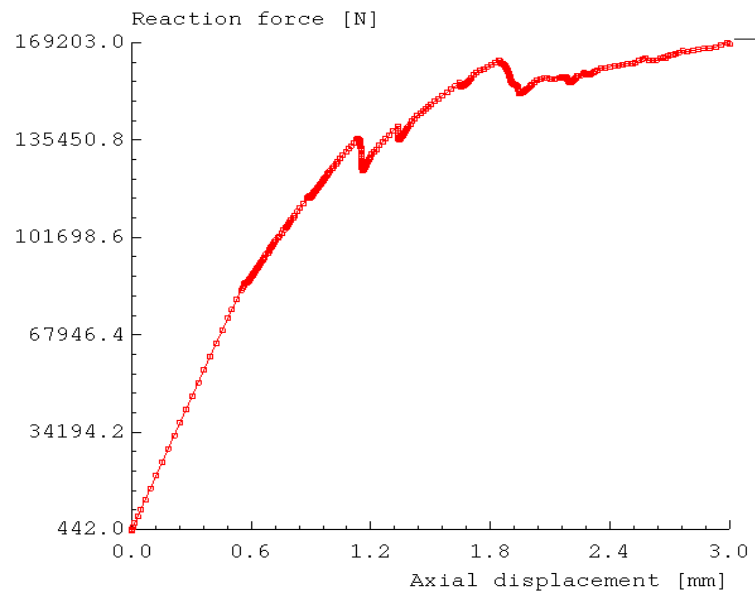
In the post-buckling region the transient solver is more accurate and robust than the non-linear static solver.



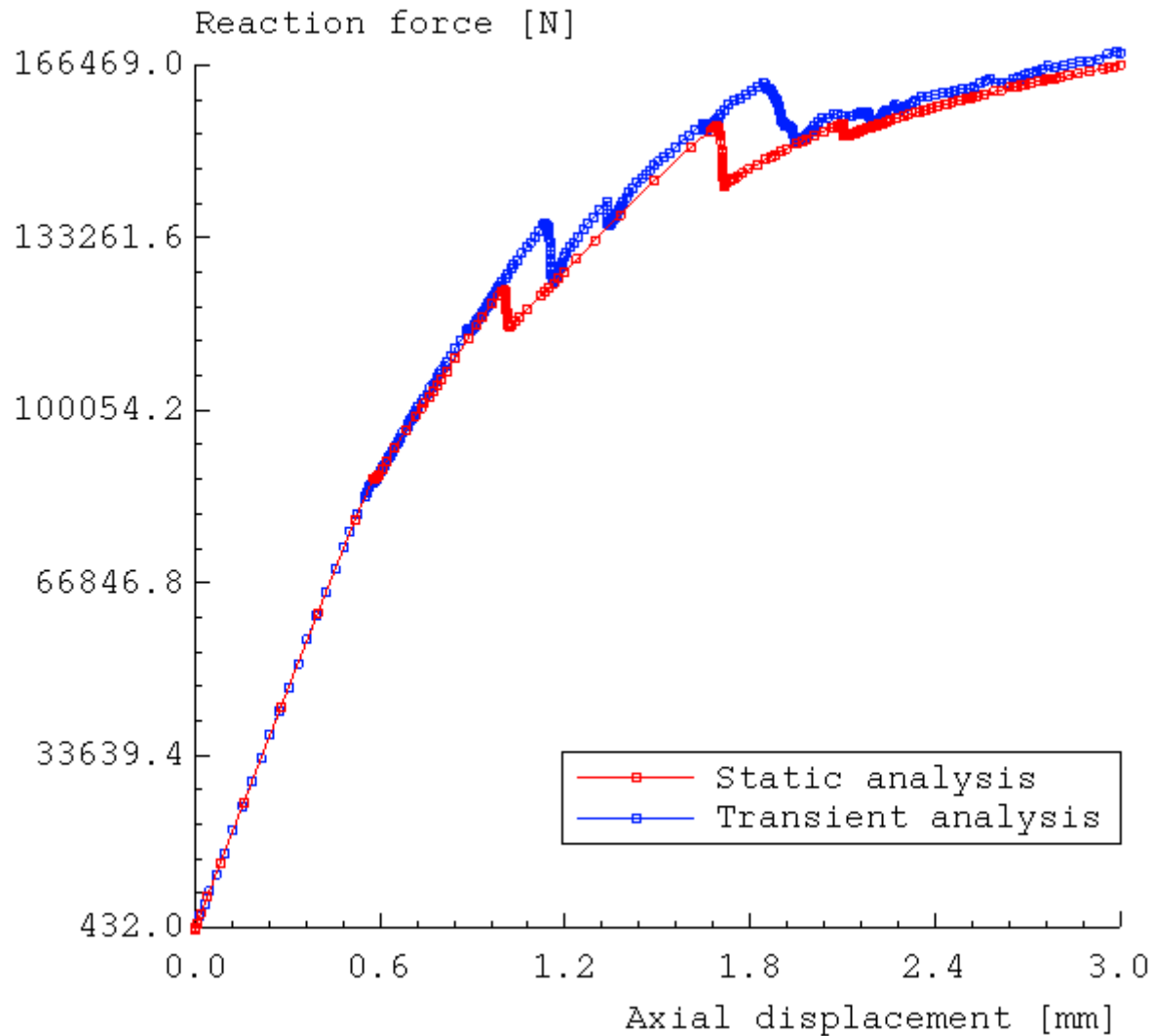
# The B2000 implicit transient solver

Post-buckling analysis results with the PSC5 benchmark using the Q4.MITC\_E element:

- First local buckling at 0.58 mm and 86 KN
- First global buckling at 1.13 mm and 136 KN
- Computation time on an AMD Athlon 3500+ Linux-PC: **419s!**



# The B2000 implicit transient solver



## Comparison between static and transient analysis:

- The limit load of the transient analysis is higher than that of the static analysis (the transient analysis takes a different but still symmetric path – due to non-symmetric imperfections, the solution is different from the static one).
- Although the transient analysis needs more steps than the static analysis, the time to solution is much shorter because the transient problem is better conditioned and thus requires fewer re-calculations of the second variation.



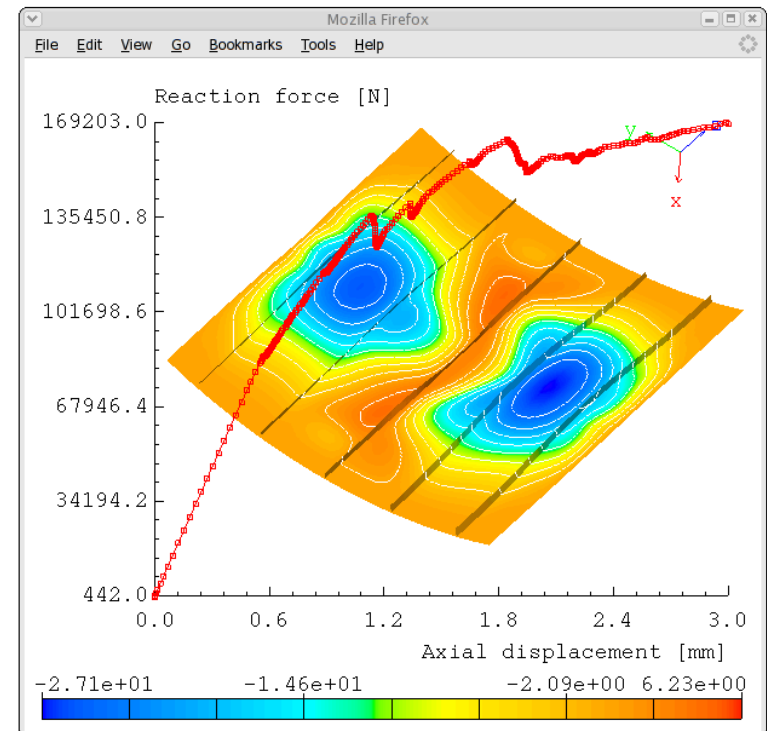
# B2000 video demonstrations

The video sequence *dynamic.gif* included in this directory demonstrates the evolution of the non-linear transient calculation of a compressed composite panel:

- The panel is compressed in axial direction with constant velocity.
- The material remains elastic.
- Observe the automatic adaptive time step!

Instructions for viewing:

- Under Linux, launch with firefox, mozilla, or konqueror (from a shell with `firefox `pwd`/dynamic.gif`).
- Under Windows, open *dynamic.gif* with the Windows Media Player.



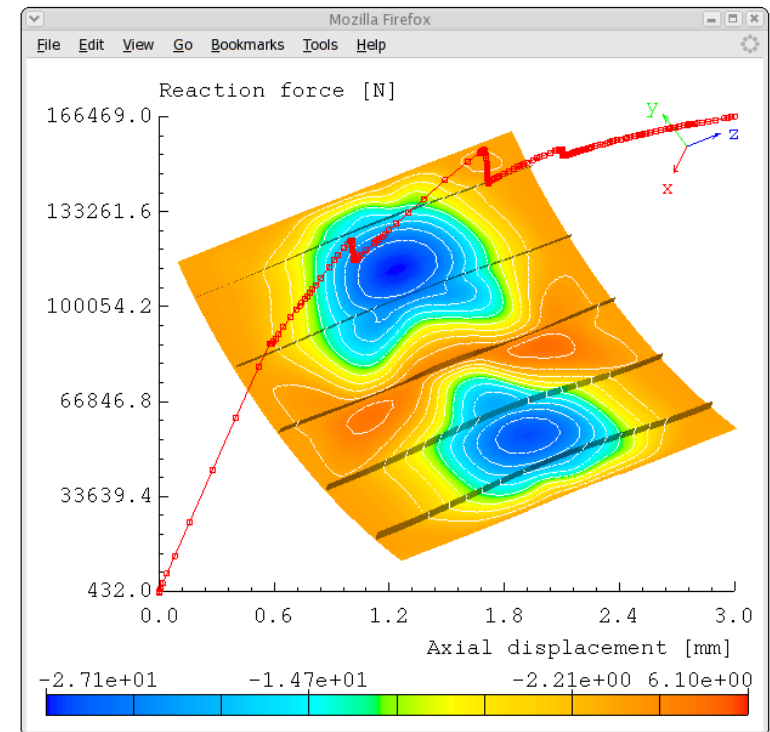
# B2000 video demonstrations

The video sequence *static.gif* included in this directory demonstrates the evolution of the non-linear static calculation of a compressed composite panel:

- The panel is loaded statically by applying an axial displacement.
- Where necessary an 'artificial damping' procedure is applied to overcome critical regions.
- The material remains elastic.

Instructions for viewing:

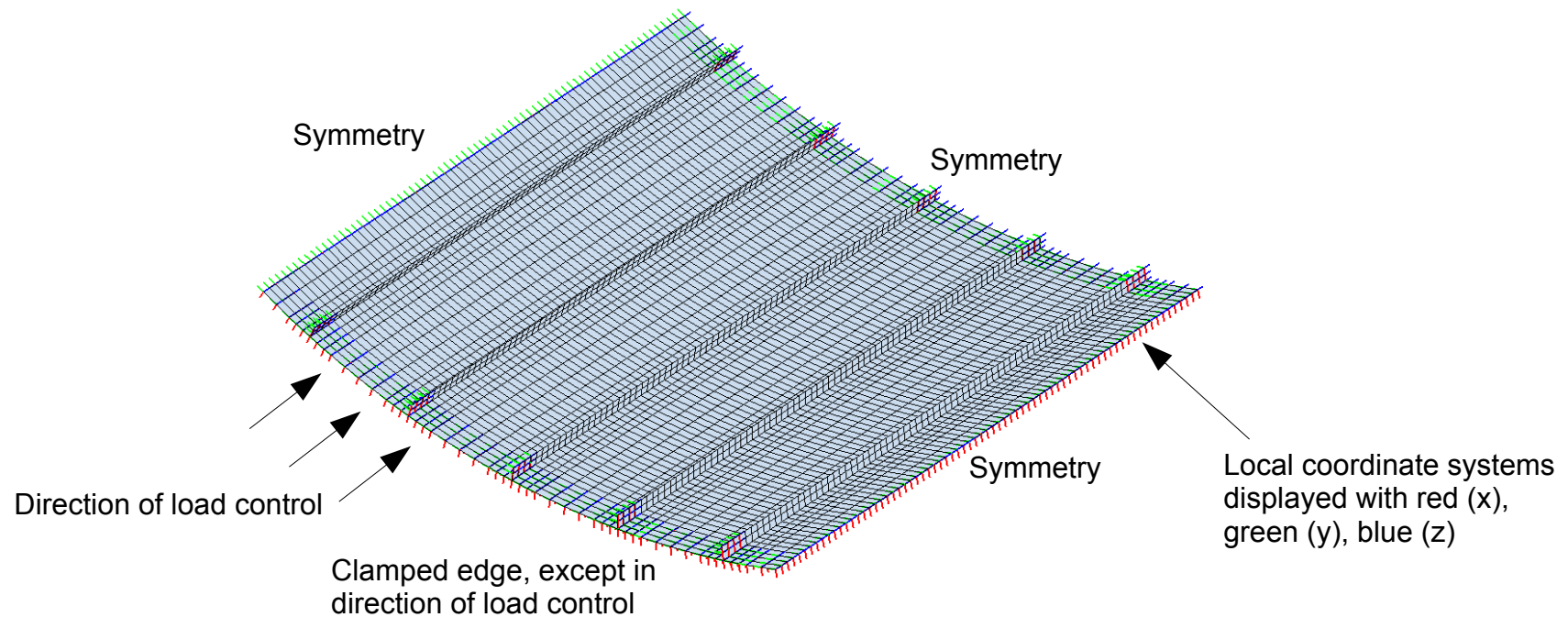
- Under Linux, launch with firefox, mozilla, or konqueror (from a shell with `firefox `pwd`/dynamic.gif`).
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# B2000 demonstration

A demonstration version of B2000 is available for the **x86 architecture running Linux Fedora Core 5.**

The demonstration runs with one of the typical COCOMAT panels exhibiting imperfections.



Mesh and boundary conditions of COCOMAT demonstration case



# B2000 transient analysis demonstration

The non-linear transient calculation of a compressed composite panel is similar to the one displayed in the *dynamic.gif* video sequence:

- The panel is compressed in axial direction with constant velocity such that an axial displacement of 3 mm is reached after 100 s.
- The material remains elastic.
- The time step is determined automatically.

The video sequence can also be viewed from <http://www.smr.ch/misc/projects> (section COCOMAT).

For instructions on how to run the demonstration version of B2000 please read the *REAME* file on the demo CD.



# B2000 static analysis demonstration

The static calculation of a compressed composite panel is similar to the one displayed in the *static.gif* video sequence:

- The panel is compressed in axial direction from 0 to 3 mm.
- The material remains elastic.
- The continuation procedure is fully automatic.
- Where necessary an 'artificial damping' procedure is applied to overcome critical regions.

The video sequence can also be viewed from <http://www.smr.ch/misc/projects> (section COCOMAT).

For instructions on how to run the demonstration version of B2000 please read the *REAME* file on the demo CD.

