Tools for the evaluation of the residual strength of cracked pressurized fuselage shells

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Summary

This paper discusses some computational tools, recently developed, that are useful in the assessment of the residual strength of fuselage shells damaged by through cracks.

1. Introduction

Fuselages can accumulate damage in the form of through cracks in the outer containment shell. The prediction of the way these cracks propagate under service loads is therefore an important issue in the assessment of the residual strength of such a shell. This paper is focused on some of the tools of analysis recently developed that are capable to assess the effect of this type of damage.

A through crack in a pressurized shell is known to be dominated by the following influence factors: (i) the curvature, which is the principal reason the crack faces bulge out; (ii) a geometrically nonlinear effect that usually, but not always, alleviates crack tip loading[1]; and, (iii) the ductility of the material, so that a plastic zone develops in front of the crack tip and absorbs a part of the energy that must be supplied to force the crack to advance.

The tools for analysis of the way cracks behave under in-flight loads must therefore adequately represent the mechanics of the problem described above. In addition they should be capable of providing insight in the way the crack can grow under influence of service loads, i.e., internal pressure plus the additional loads that are induced by the in-flight conditions such as maneuvering and/or gust loads [2-8].

In this paper we give an short survey of some of the features of the STAGS code[9] that enable us to simulate crack growth under a slowly increasing load. It includes references to the background of the shell equations, the plasticity model and the node release method that lie at the basis of the whole complex of methods that is used to solve this problem. It also includes the solution method that is used to determine the “load versus crack (path) length” diagram from which the residual

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strength of the damaged structure can be deduced [10]. We will begin this survey with a short introduction to the mechanics of the “cracked fuselage” problem.

2. The basic mechanical features of the problem

The bulge factor

The stress distribution in a thin-walled (unstiffened) infinitely long cylindrical shell under internal pressure is given by:

$$\sigma_x = pR/2t; \quad \sigma_y = pR/t \qquad (1)$$

where p is the internal pressure, R is the radius of the shell and t its thickness. It is immediately noticed that in this case the hoop stress σ_y is twice as large as the axial stress σ_x.

If stiffeners are added to the shell as is done in airplane fuselages, the stress distribution is no longer uniform and the peak values of the stress components σ_x and σ_y are reduced as compared to the situation in the unstiffened case (at the same value of the pressure p). But for the hoop stress σ_y this reduction is only marginal (the order of magnitude of about 12%, except in the neighborhood of the bulkheads of the shell where the reduction can be larger). The large hoop stresses in comparison to the other components is the reason that cracks in pressurized fuselage shells often tend to grow in longitudinal direction (Figure 1).

If we could observe a crack in a fuselage while the cabin pressure is applied, we would be able to see that the crack faces bulge out. It is often thought that this type of deformation is caused by the internal pressure itself, that, as a distributed set of loads normal to the shell, tries to force the crack faces to open. But in reality, this effect is only marginally present. The real reason for the crack to bulge out is a geometrical effect, i.e. the curvature of the shell.

To understand this effect properly it is useful to conduct the following thought experiment. To compute the stress distribution in a thin pressurized cylindrical shell with a longitudinal crack, we could try to obtain the solution in two stages. In the first stage (i) the crack is artificially kept closed by an appropriate set of constraints and we let the pressure p increase from zero to the desired nominal value. The stress state in the model will then be given by (1). In the second stage of the computation (ii) we slowly release the constraints, i.e., the forces that were necessary to keep the crack closed during the first stage of the computations. This system of forces along the crack faces in fact correspond to the hoop stresses σ_y acting on both sides of the crack edges at the end of stage (i).

The release process in stage (ii) is thus similar to adding a load system to the cylinder in the situation reached at the end of stage (i), that consists of a stress distribution $\sigma_y = pR/t$ along the crack edges with opposite sign as is shown in Figure 2. It now becomes immediately clear that this annihilating edge loading will force the edges to bulge out as is illustrated in the figure.

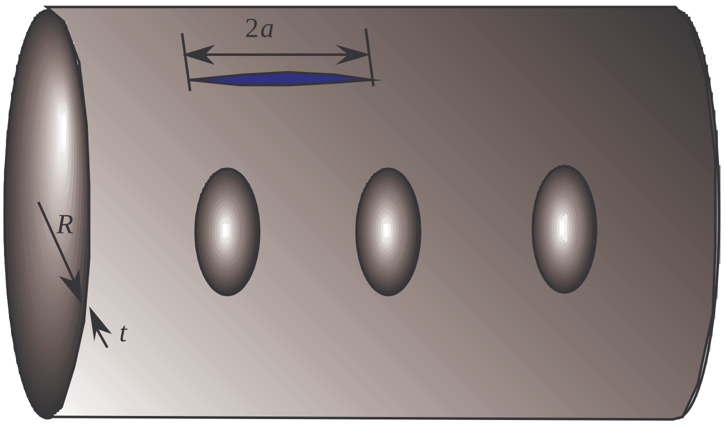


Figure 1 Fuselage shell with longitudinal crack

The opening of the crack as the result of this additional force system cannot be resisted by membrane stiffness of the shell alone. It must also be resisted by the bending stiffness of the shell and this stiffness is relatively small as a result of the high radius to thickness ratios[&] that we are dealing with here. The meaning of all this is that the crack opening angle in the cylinder loaded in the far field by $\sigma_y = pR/t$ will be larger than the crack opening angle of an identical crack in a flat plate of identical thickness and material properties. This amplifying effect in the crack tip loading condition is called bulge effect. For a given shell with a certain R/t ratio the effect will more important the longer the crack is. The parameter that is believed to be determinate for this effect is [11]:

$$\alpha = 2\sqrt[4]{12\left(1-\upsilon^2\right)}\sqrt{\frac{a^2}{Rt}} \qquad (2)$$

Geometrical nonlinear effect

The bulge effect can be brought into perspective by a linear elastic solution of the crack tip stress field [11]. It turned out that for small cracks, this linear solution represents the stress distribution in the sketched situation fairly well, but for longer cracks the solution becomes hopelessly inadequate. The reason for this phenomenon is that there is a strong geometrical nonlinear mechanism that has - in particular for longitudinal cracks - a very large effect on the crack tip loading [1].

To explain the nonlinear effect, we point out that the bulging of the crack faces must be accompanied by an elongation of the material fibers aligned along the crack faces (see Figure 2). It is to this type of stretch that the shell puts up an enormous resistance. It is this stiffening mechanism that lowers the crack tip stress intensity appreciably as compared to the linear solution.

[&] For current airliners the R/t ratio is about 1800.

The crack edge opening displacement as a function of pressure according to the linear and nonlinear theory illustrates this observation quite clearly (see Figure 3).

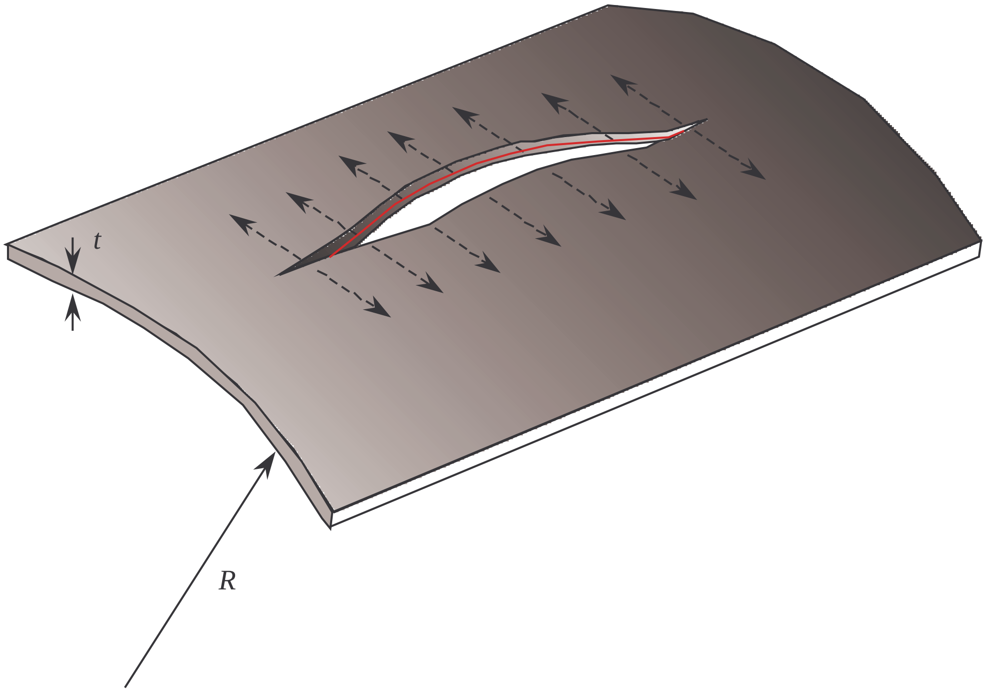


Figure 2 The bulge effect

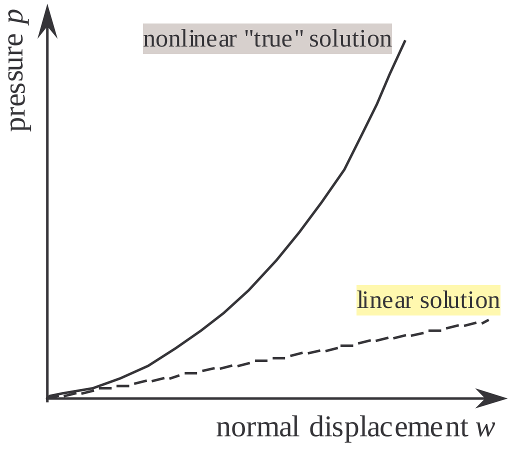


Figure 3: Crack edge bulging displacement according to the linear and nonlinear theory

This resistance to further opening is especially important if the axial stress σ_x is positive as in the loading case we considered above. On the other hand when the axial stress σ_x becomes negative the beneficial effect on the crack tip loading is reversed. The latter situation may occur in some parts of the fuselage because, next to internal pressure, there are also other type of in-flight loads acting on the airframe. Consequently, geometrical nonlinear effects play an important role in the behavior of through cracks in fuselage shells so that we believe that it is not permissible to ignore these effects in the formulation of residual strength models presently under discussion.

Cracktip plasticity

The skin of most present-day aircraft fuselage construction is made of ductile aluminum alloys. We will assume here that skins of this material are homogeneous and initially isotropic (although we know that because of the manufactoring process this assumption can only be approximately valid). The ductility of the material is the reason that a plastic zone develops around the tip of a crack when the structure is loaded. During the formation of a new fracture surface this plastic zone moves and expands with the crack tip, a process that is accompanied by energy dissipation. The energy dissipation is the key to the explanation why a crack is able to grow under an increasing load in a controlled manner. By "controlled" we mean that it can grow stably for some range of the loading process [10]. The residual strength of cracked skins of these materials is thus strongly influenced by the elastic-plastic material properties and this means that these properties must be incorporated in the formulation of the numerical model in an appropriate way.

Three dimensional representation of the crack tip stress field

The behavior of the fuselage skin and its supporting stiffeners, bulkheads etc. can conveniently and adequately be modeled by a shell theory that is based on the assumption that the state of stress in the shell is approximately two dimensional, i.e. a state of plane stress. When a crack is present in such a structure the plane stress assumption is still valid for the greater part of the structure but no longer in the immediate neighborhood of the crack tip(s). At these regions this assumption is clearly violated.

In many engineering applications this difficulty is circumvented by assuming that the modification of the two-dimensional plane stress model to a three-dimensional formulation does not lead to a substantial differences of the simulation of crack behavior. After all, the modification of the stress and deformation field when going from the first to the second stage takes place in a very small part of the structure in the immediate area surrounding the crack tip. This volume has an average diameter of no more than one or two times the wall thickness of the shell. Moreover, it is believed that the behavior of the crack is principally governed by the membrane state of stress, and not by the bending part of the solution. This membrane state can be adequately described by shell theory. Most residual strength evaluations to date are based on this simplified approach.

On the other hand, in view of the flexibility that finite element modeling offers, it is not necessary to restrict oneself to the exclusive use of shell theory. One can also apply a full three-dimensional formulation. To do this uniformly, i.e., for the whole domain that the structure maps out would in general be prohibitive, but to do this in a selected region where it is most needed is an entirely feasible undertaking. In this paper we will make use of such approach, i.e., we will use a substructuring technique whereby the anticipated fracture path and its neighborhood is modeled by three-dimensional finite elements. With this refinement, we are able to make an assessment of the error that is made if the analysis is restricted to a two dimensional approach only, and, it may also reveal some interesting aspects of the crack propagation process through the thickness of the shell.

3. Some useful features of the STAGS code

Shell finite element, the basic building stone

The STAGS code is equipped with triangular and quadrilateral shell elements that obey the plane stress assumptions and that are formulated in a corotational framework [9,12,13]. This renders the solutions valid in the range of arbitrary displacements and rotations with the only restriction that the strains remain small (< O(0.01)). Models of sections of the fuselage can be built by these elements allowing a large degree of detail, including lap joints, stiffeners and bulkheads. Examples of shell models created by this code can be found in for example[2-8].

Volume elements

The state of stress and strain in the immediate surroundings of the crack front in a shell is three-dimensional and cannot be simulated by shell elements that only represent plane stress states. As mentioned, this difficulty can be circumvented by a refinement technique that uses three dimensional volume elements. The elements in STAGS that are applied for this purpose are standard eight noded isoparametric brick elements [9]. For economy reasons they are only introduced in a boundary layer of the current and prospective crack fracture surfaces.

A detail of interest is the way the volume elements are connected to the surrounding shell elements. In STAGS, the principle that is followed is described in [14].

The juncture of the substructure volume elements to the surrounding body of the shell elements must satisfy two constraints. First, the weighted average through the thickness of the solid node displacements must match the corresponding shell "master node"; second, each of these nodes are restricted to move only *along the shell director*. This latter constraint complies with the basic assumption of shell theory that normals stay normal, and that the strain is a linear function of the thickness. A consequence of this constraint is that the bounding top and bottom surfaces of the shell and solid substructures are free of normal stress in the neighborhood of the juncture line.

Plasticity model

We use the fraction model that was originally proposed by Besseling[15]. In this model, the state of stress of a material point of the body under investigation can be subdivided in components (called fractions), each of which possesses its own yield function and flow rule. The state of strain of each fraction is identical to that of the material point itself. In STAGS the individual fractions are treated as elastic-perfectly plastic whereby each fraction has its own yield stress and yield strain. The resulting model is capable of representing cyclic straining.

To integrate the plastic stress for the computation of the nodal force residuals, STAGS uses the well-known radial return method. A comprehensive description of this method and its implementation can be found in [16]. STAGS uses a slight modification of what is presented in the reference just mentioned: we divide the strain increment developed between two load steps into subincrements whose size is determined by the ratio of the predicted elastic stress increment and the yield stress. Each increment must produce no more than 20% overshoot in predicted stress. Thereafter the procedure described in [16] is adopted for each subincrement until all of the new fraction stress states are computed.

Crack growth simulation

In the residual strength analyses of the kind discussed here we will simulate crack growth in a self-similar fashion; i.e. along a predetermined fracture path if the cracked structure is modeled by shell elements only. In this case the nodal release method lends itself as a very useful and practical tool [10,17]. The crack growth that we try to compute is supposed to take place under variable load. More precisely: the simulation tries to predict what happens when a pre-cracked structure is stressed by a slowly rising load, a loading process that is supposed to take place in a quasi-static fashion. The special point of interest in this computation is the determination of the load and the corresponding crack length at which crack growth becomes unstable (and the quasi-static process turns into an transient process).

The solution of the governing equations

The problem of the growing crack is determined by the equilibrium equations for the structure as a whole with the crack growth criterion a side condition. The path following method that is employed for the solution of this extended set of equations is described in [10]. In this procedure the nodal release method—mentioned earlier—plays a dominant role. Care is hereby taken that the solution obtained is in agreement with the physics of the fracture growth process as best as possible by trying to avoid spurious path dependent effects.

Aspects of the solution when volume elements are used

In reference [10], crack growth simulation is described for the case that the crack is modeled with two-dimensional shell elements. In that case, the crack front is fixed by a single pair of master and slave nodes. If we employ a three-dimensional discretization of the crack surroundings, the crackfront is determined by a string of double nodes (2N+1 in total) which lie along an, initially, straight line in the thickness direction of shell. In what way is the nodal release method carried out in this case?

As it has turned out there is really no great difference between the node release in solids as compared to node release in shells. Effectively, the model can now be viewed to have 2N+1 *independent* cracks, instead of one, each with its master and slave nodes. Thus, as far as STAGS is concerned, the crack front could just as well be seen as 2N+1 cracked shell layers packed on top of each other instead of 2N+1 layers of volume element nodes.

For the crack growth criterion, an adaptation of the well-known Crack-Tip-Opening-Angle criterion is used. This criterion concerns the maximum dihedral angle along the crack front at the location in question. But since the layers are very thin here, it suffices to measure the CTOA simply by taking the displacement difference of a pair of nodes in the wake of the crack front and compute the enclosed angle with respect to the frontal node. In this way we are able to get entirely satisfying results, as we will demonstrate at the end of this paper.

As far as the energy release rate components are concerned, they are computed just as before [17], but only the *sum* of them across the thickness is really meaningful. We do, however, allow nodes to be released independently, so that tearing can begin at the top surface. It should be stressed however, that in the cases we studied here, once a given node is released in the thickness direction, all the others follow. This means that with the resolution applied in section 4 (four volume element layers through the thickness) a real tunneling effect was never observed. We believe that this is due to the circumstance that for finding such an effect it is necessary to apply a model with a considerably denser mesh. We will return to this topic in the next section.

4. Examples and Conclusion

A particular example

The subject of this paper is crack growth simulation in pressurized fuselages. It would therefore seem natural to conclude this short overview with an example that involves a fuselage section or at least a cylindrical shell segment. That we do not opt to do this here is because we prefer to demonstrate the growth simulation procedure on a flat plate specimen for which laboratory test results have been obtained. Test on curved panels that mimic the conditions in a real fuselage under pressure are very difficult and expensive to conduct and are therefore rare. For studies on fuselage segments or cylindrical shells where STAGS is used as simulation tool we refer to [2-8].

The test article that we introduce as an example is taken from a test program carried out by NASA as reported in [18]. This report contains an extensive computational validation of the test program carried out with complete 3D finite element models. The overall geometry of the specimen - a centrally cracked plate- that we selected from this program is pictured in Figure 4 while its material properties are given below:

(i) Young’s modules and Poisson’s ration at zero strain $E = 10.35\ E^6$ psi; $\nu = 0.3$ respectively.

(ii) The stress strain curve used as input for the plasticity model is given in piecewise linear form with the data points (σ, ε):

(50. E^3 psi, .000483)
(56.6 E^3 psi, .015000)
(62.3 E^3 psi, .040000)
(68.2 E^3 psi, 1000000)

Using the symmetry of the test setup only one quart of the panel is modeled. In [10] we computed the load versus crack length diagram for this panel with an “all” shell element model. In the present paper we refined the crack region of this model with volume elements in the way described earlier. Our choice of mesh for the refined model is pictured in Figure 5^a while the details of the mesh refinement along the crack path region is shown in Figure 5^b. The calculations are here carried out for a prescribed load uniformly distributed along the transverse edges. This differs from the test conditions where the edges were clamped and the loading was introduced by controlling the plate end displacement. However, the difference between the edge conditions for our model and those of the actual experiment has a negligible effect on the final results because the loaded edges are far removed from the crack.

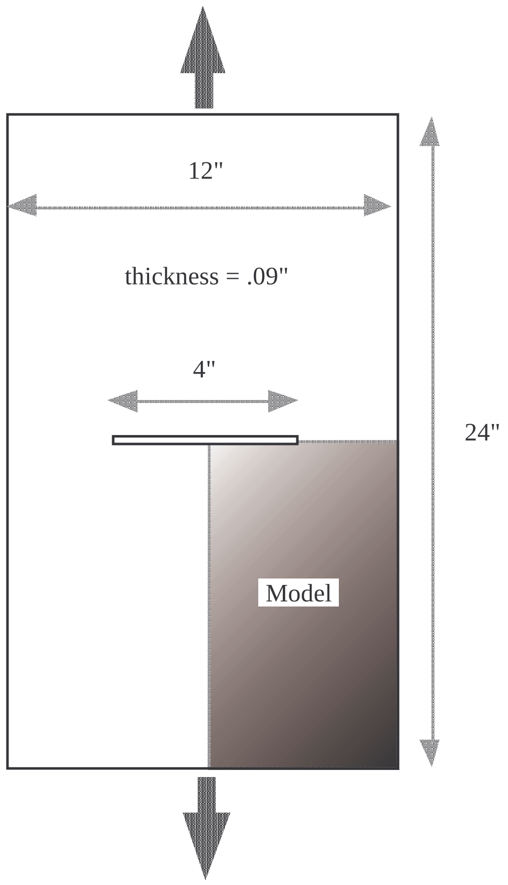
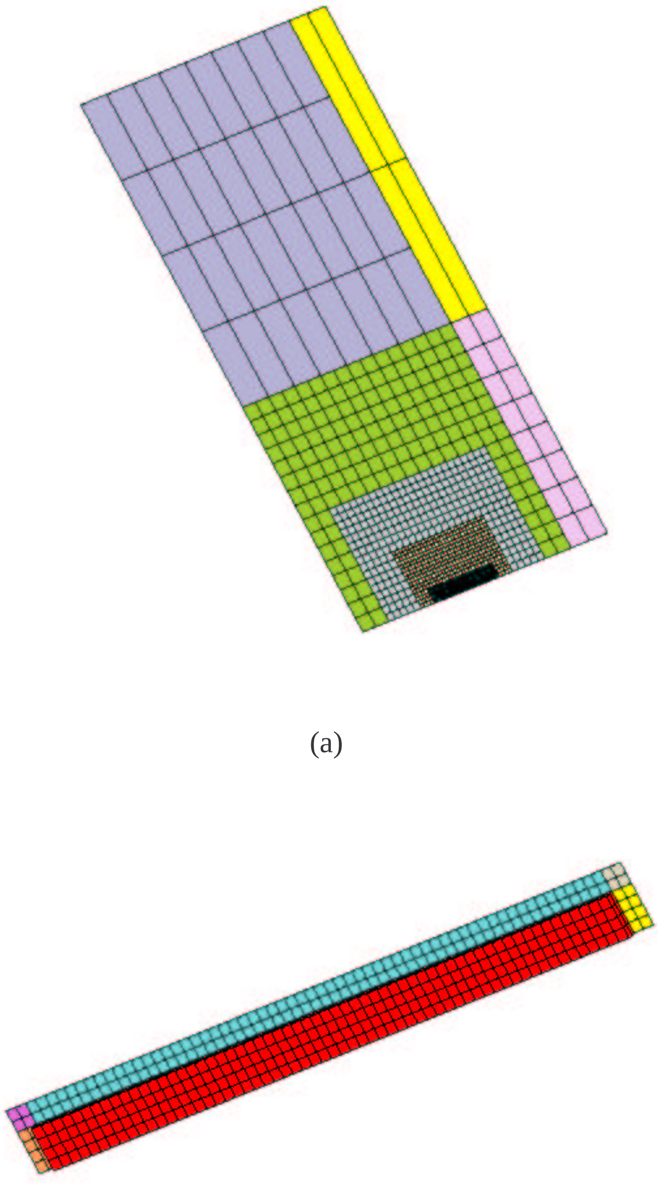


Figure 4 Test article

We should emphasize that the test program, as well as the analysis, covers two different test cases. This difference is related to the desire to discern the effect caused by the tendency of the crack region to buckle out when the panel is pulled (refer to [19] for an analysis of this phenomenon). This means that in one series of tests buckling was prevented, while in the other series, buckling was allowed to take place. The results we present here cover both cases.

All calculations were carried out with the same step length δa = .04in. We used the CTOA[&] criterion with the value of the critical opening angle given as: $\alpha = 5.5°$.

[&] Refer to18 for an discussion and justification of the use of this criterion in this particular case



(a)

(b)

Figure 5^{a+b} STAGS model of the plate

The results of our calculations are summarized in Figure 6 where they are compared with the test data obtained in the NASA test program. As can be seen from the picture the agreement between the simulation and the actual test data is quite satisfactory, in particular if we take into account the fact that the mesh applied here is rather coarse as compared with the meshes employed in [18].

How do these results compare with our previous computations conducted with a traditional *plane-stress* shell finite element model? It turns out that the plane stress representation of the crack tip stress field causes us to substantially overestimate both the critical crack length and load, a tendency that is particularly strong when bending occurs at the crack tip (the case without buckling guides). We believe that the reason for this error is that the plane stress constraint imposed on the crack tip stress field represents an unreasonable stiffening effect that hinders the opening of the crack faces at the tip. Although this effect on the crack tip opening displacements is relatively small as long as the problem is treated purely elastically, it turns out that it becomes much more pronounced when small scale yielding is allowed to take place.

Conclusion

We discussed some of the tools implemented in STAGS that are useful in the evaluation of the residual strength of fuselage shells under pressure and in-flight loads. In our demonstration we focused in particular on the use of solid elements to model the crack region of the shell so that, at least to some degree of approximation, the three dimensional effects that play an important role in the growth mechanism are accounted for.

It is true that in the examples that we tried out so far, the crack front did not change in profile through the thickness while it was advancing from the initial length a_0 to the final value a_e. In other words, the crack front remained effectively straight. Only during the release process itself, i.e., during a step δa, the CTOA criterion would be met first in one particular layer of nodes before the others, but upon release of the corresponding node the remaining ones would invariably follow. As expected, in the cases where buckling was allowed to take place, the first node to be released was always at the outside of the shell where the bending stress was at its peak.

The fact that we have not yet observed tunneling of the crack front profile must be due to the coarse mesh that was applied in these examples. This can be gleaned from [18] where the computation of tunneling effects were studied in models with meshes that are considerably more refined than ours (meshes with something like 28 volume elements through the thickness of the shell). It is clear that more experimentation with our model and procedure is required before we can expect to make a definite observation regarding these phenomena.

On the other hand, the authors of [18] argue that the tunneling effect plays a minor role in crack growth in thin shells of the t/R ratios studied here. This argument is further supported by the excellent results that were obtained with their (simplified) frontal release approach where all crack front nodes are advanced simultaneously. The growth simulations that we computed with our model so far is actually very similar to this frontal release procedure, and the results obtained by it seem thus to confirm their observations. It is finally noted that the model we displayed here requires far less computation than the models used in [18], a factor very important in practical applications.

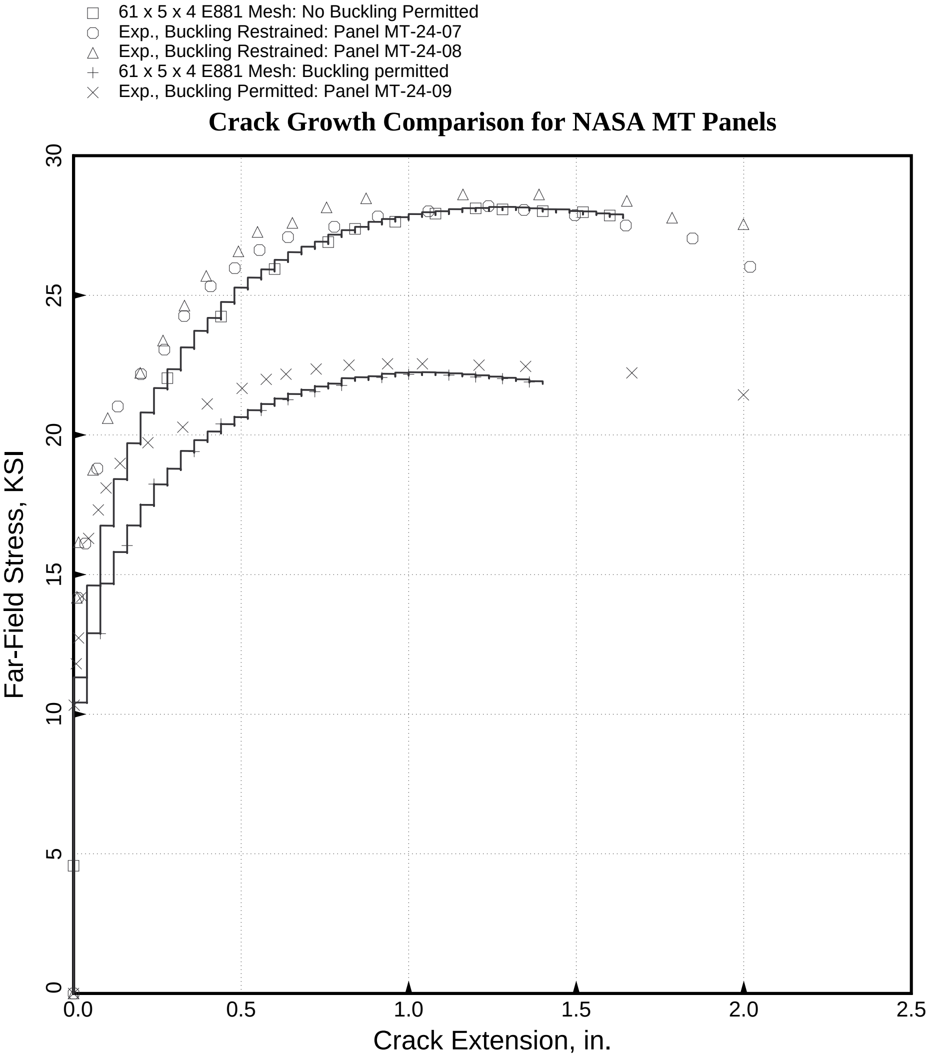


Figure 6 Test data and STAGS results

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