1. I Cross Section Beam with Shell Elements

A beam with an I cross section (HEA320 wide flange profile, S235JR EN10034/EN10025) is meshed with Q4 shell elements patches, an upper and a lower flange patch and a web patch. The beam is clamped at both ends. Due to symmetry in loading and boundary conditions, one half of the beam is modelled. The beam is loaded with a 'concentrated' force at the beam mid-span, i.e. a the right end of the FEM model displayed in the Figure 1 below. Two models are tested:

- A continuous mesh model, where the element of the 3 patches match and where the patches are connected with the *MDL* **join** command.
- A discontinuous (non-matching) mesh model, where the 3 patches are 'tied' together with the common mesh refinement kinematic coupling method through the *MDL* **field_transfer** command.

Both meshes have a similar mesh density. The analysis is linear.

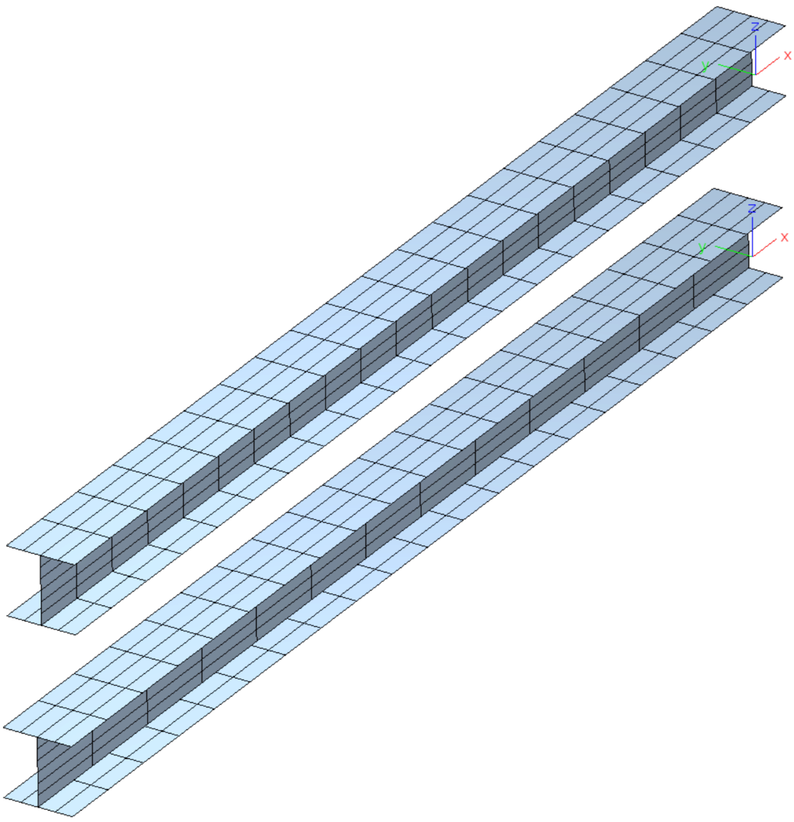

Figure 1. HEA profile with shell elements: Mesh (upper model: continuous mesh, lower model: discontinuous mesh).

The resulting deformed shape for the continuous mesh and the discontinuous mesh as well as the longitudinal stresses are show in Figure 2.

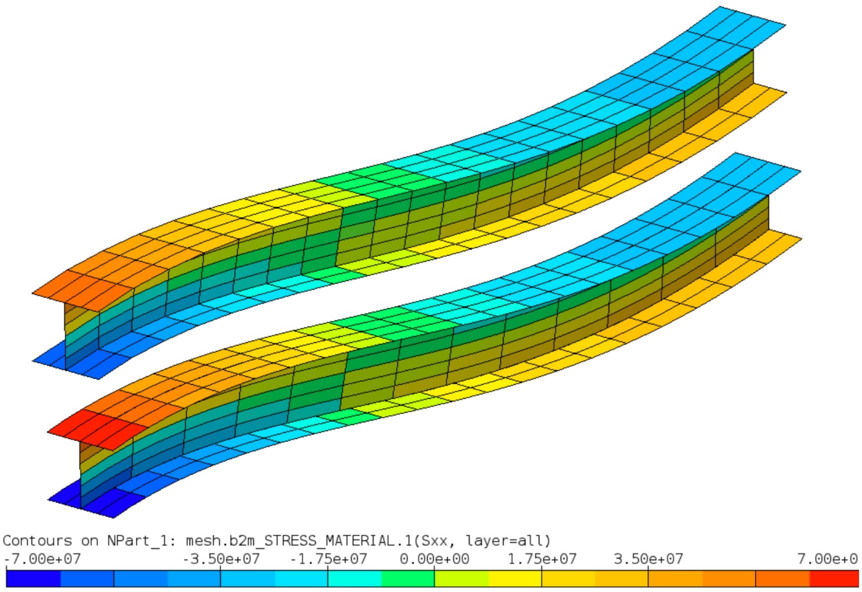

Figure 2. HEA profile with shell elements: Amplified deformed shape and longitudinal stresses (upper model: continuous mesh, lower model: discontinuous mesh).

The analytic reference solution (Euler beam deformation) is readily obtained from the literature:[1]

$$d_{\text{mid-span}} = \frac{PL^4}{EJ},\ M_{\text{clamped}} = \frac{PL^2}{12},\ M_{\text{mid-span}} = \frac{PL^2}{24}$$

The table below resumes the solutions for a beam model and the shell models:

Model	$d_{\text{mid-span}}$	σ_{clamped}	$\sigma_{\text{mid-span}}$
Analytic	0.0613	63.8	31.9
Shell continuous mesh	0.0613	60.2	30.6
Shell tying	0.0613	69.8	31.7

The mdl input file for the shell case with tying is listed below. Comments and explanations are included in the input file.

```
title 'HEA320 shell model, tying, units: SI'

# Parameters
(eltype?="Q9.S.MITC")
(t?=0.0155)          # Flange thickness
(tsteg?=0.009)       # Web thickness
(h?=0.31)            # Height
(w?=0.3)             # Width

(ne1_flansch?=20) # N. of elements flange in x
(ne2_flansch?=4)  # N. of elements flange in y
(ne1_steg?=13)    # N. of elements web in x
(ne2_steg?=6)     # N. of elements web in y

# Derived parameters
(w2=w*0.5)
```

[1]R. J. Roark, W. C. Young, Formulas for Stress & Strain, Sixth edition, McGraw-Hill Book Company (1975).

```
(h2=0.5*h -0.5*t)
(l=5.)

epatch 1
    title 'Mesh of Bottom flange'
    geometry plate
    p1 0 (-w2) (-h2)
    p2 (l) (-w2) (-h2)
    p3 (l) (w2) (-h2)
    p4 0 (w2) (-h2)
    eltype (eltype)
    ne1 (ne1_flansch) ne2 (ne2_flansch)
    mid 1
    thickness (t)
end
epatch 2
    title 'Mesh of top flange'
    geometry plate
    p1 0 (-w2) (h2)
    p2 (l) (-w2) (h2)
    p3 (l) (w2) (h2)
    p4 0 (w2) (h2)
    eltype (eltype)
    ne1 (ne1_flansch) ne2 (ne2_flansch)
    mid 1
    thickness (t)
end
epatch 3
    title 'Mesh of web'
    geometry plate
    p1 0 0 (-h2)
    p2 (l) 0 (-h2)
    p3 (l) 0 (h2)
    p4 0 0 (h2)
    eltype (eltype)
    ne1 (ne1_steg) ne2 (ne2_steg)
    mid 1
    thickness (tsteg)
end

material 1
   type ISOTROPIC
   title 'Steel'
   density 7800
   E 21.e9 P 0.0 alpha 23.0e-6
end

nbc 1
   type line_loads system branch
   title 'Concentrated load at mid-span disitributed along width'
   line_loads 0  0 -10767.456
   epatch 3 e3
end

ebc 1
   title 'Left edge: Clamped'
```

```
   value 0.0 dof [1 2 3 4 5 6] epatch 1 e4 epatch 2 e4 epatch 3 e4
end

ebc 2
   title 'Right edge: Symmetry in y-z plane'
   value 0.0 dof [1 5 6] epatch 1 e2 epatch 2 e2 epatch 3 e2
end

field_transfer 1
  # Tie lower flange to web
  interface 1 epatch 1 f7
  interface 2 epatch 3 e1
  transfer displacement minimise_on interface 2
end

field_transfer 2
   # Tie upper flange to web
   interface 1 epatch 2 f7
   interface 2 epatch 3 e3
   transfer displacement minimise_on interface 2
end

case 1
   analysis LINEAR
   constraint_method lagrange # Required for tying!
   ebc 1            # Add constraint 1
   ebc 2            # Add constraint 1
   nbc 1            # Add concentrated load
   field_transfer 1 # Add field transfer (tying) rule 1
   field_transfer 2 # Add field transfer (tying) rule 2
   gradients 1      # Compute strains and stresses
end

adir
  cases 1 # Solve for case 1
end
```