# Global-Local Post-buckling FE Analysis with the Common Mesh Refinement Method

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**Introduction**

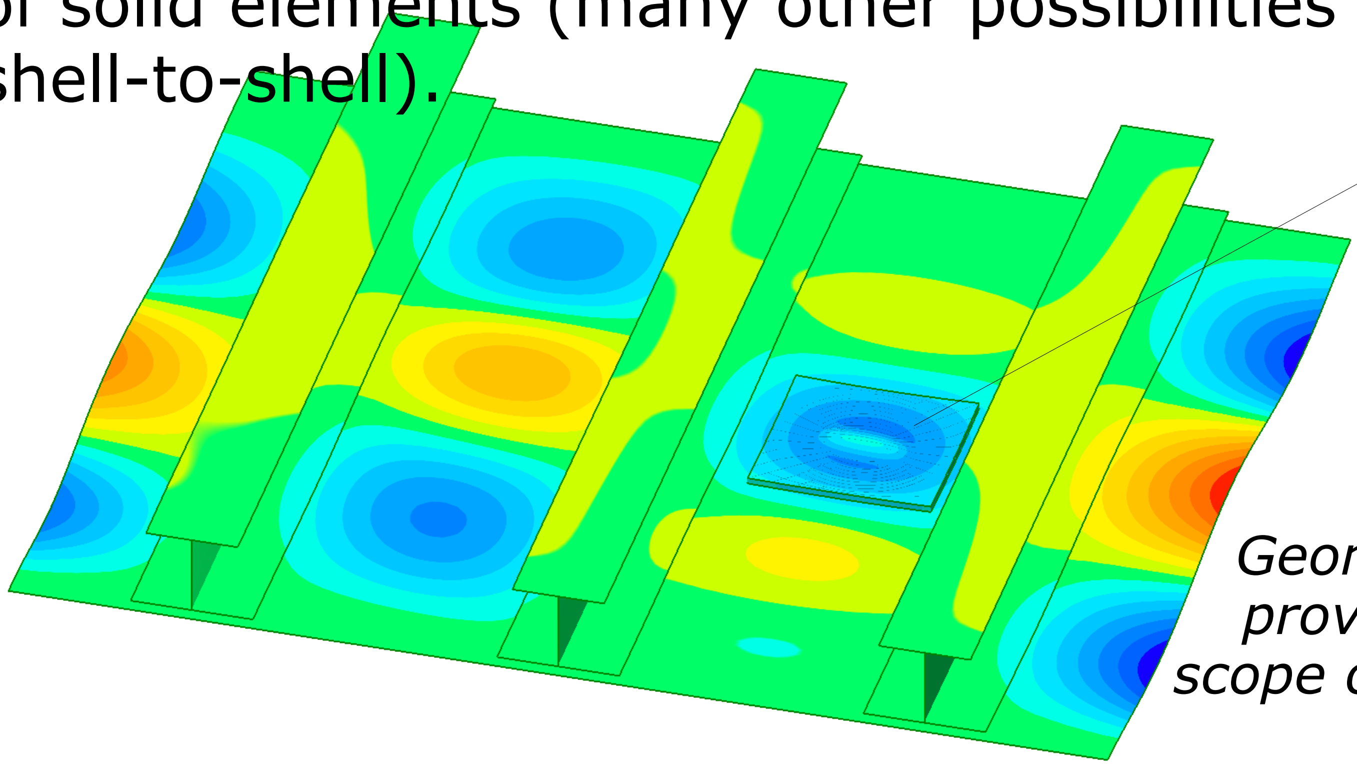
**Implementation of the common refined mesh method**

**Shell-to-solid point-wise coupling**

**Numerical Examples**

# Scope of the presented work

- Global-local analysis of composite flat or curved panels with stringers (→ kinematic coupling).
- The global FE mesh consists of shell elements.
- The local FE mesh has a high mesh density and consists of solid elements (many other possibilities exist, e.g. shell-to-shell).




*Geometry and FE model provided by CIRA in the scope of the glFEM project*

# Scope of the presented work

- The local region is - for the time being - located on the skin of the geometry.
- The local region is relatively large and is used for:
  - Delamination due to damage by impact,
  - In-ply damage onset and propagation.
- Therefore, stresses in the local region must be free from jumps and other artifacts.
- The execution time of the nonlinear analysis is dominated by the solid elements.

# Scope of the presented work

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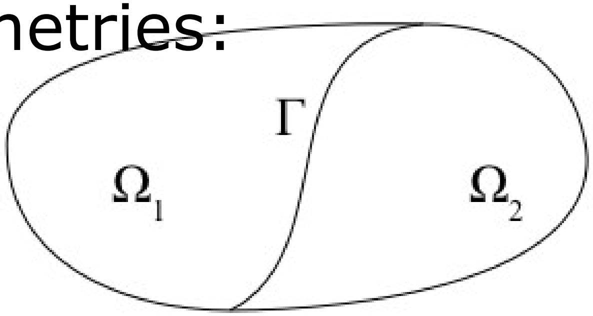
- Evaluation of the common-mesh refinement weighted-residual method against a point-wise kinematic coupling method.
- Implementation and testing is done within the B2000++ FE code, http://www.smr.ch/b2000.

# Kinematic coupling

- Source region (1) and target region (2) are discretized as separate, non-matching FE meshes but with matching geometries:




- Error of the displacements on the interface:

$$r(X) = u_1^e(X) - u_2^e(X)$$

- Formulate a set of nonlinear constraint equations (and derivatives thereof) to minimize the error (similar to rigid-body elements).
- Kinematic coupling methods are conservative but may induce stress jumps.

# Kinematic coupling

- Classes of coupling methods:
  - **Point-wise** coupling methods (point-to-surface).
  - **Weighted-residual** methods ($L_2$-minimization, surface-to-surface).
  - Others such as FETI.
- Constraints for point-wise coupling on the interface:

$$\forall i \in \{1..m\}: q_i = u_1^e(\xi_i) - u_{2,i} = 0$$

- Constraints for $L_2$-minimization:

$$\forall i \in \{1..m\}: q_i = \frac{\partial \int_\Gamma (u_1^e - u_2^e)^2 \, dX}{\partial u_{2,i}} = 0$$

# Constraint formulation and enforcement

- Constraints can be enforced using for instance:
  - Quadratic penalty method,
  - The Lagrange multiplier method,
  - The augmented Lagrange multiplier method.
- The preferred choice depends on the sparse linear solver. We use the direct, multi-frontal MUMPS solver (http://graal.ens-lyon.fr/MUMPS) with pivoting together with Lagrange multipliers.
- To avoid singularities of the second variation:
  - The constraints must be formulated such that their derivatives are nonzero everywhere.
  - There must be no linear dependency between the constraints. This requires careful enumeration of the dependent degrees-of-freedom.

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# Common mesh refinement method

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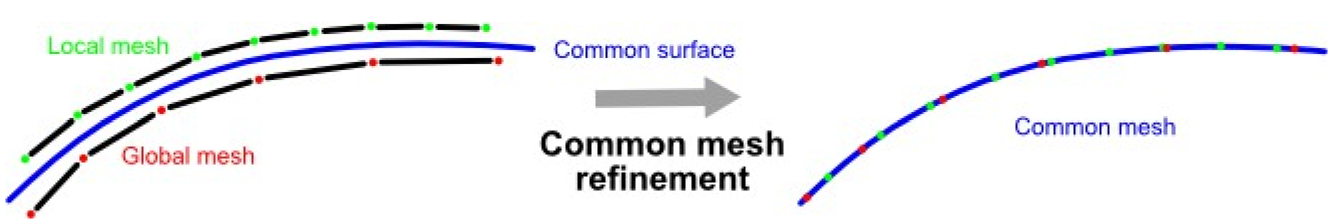
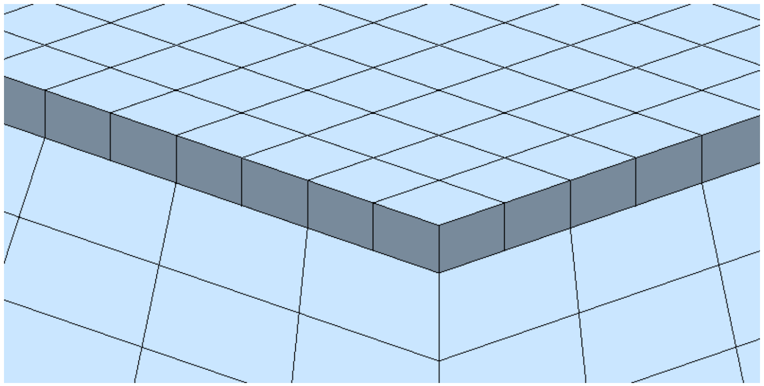
- A method to transfer fields from one surface mesh to another. Is also used for fluid-structure interaction.
- Is a weighted residual method. We choose the solid region (fine mesh) as the source region, and the shell region (coarse mesh) as the target region.
- Solves the problem of integrating over the interface.
- The matching shell and solid surface meshes are "merged" to a common mesh. Exact or nearly exact integration is performed using this common mesh.

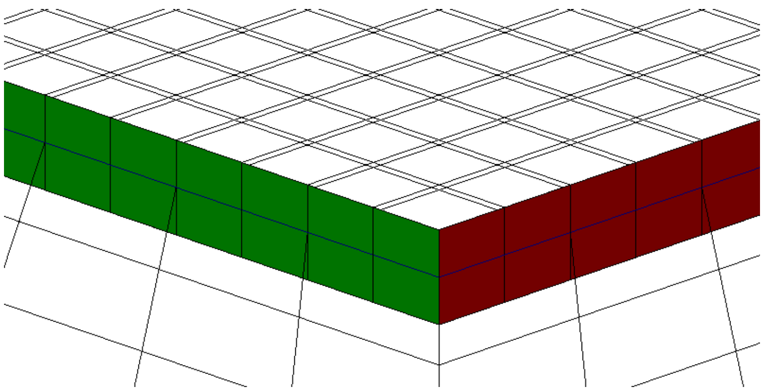
1D example:



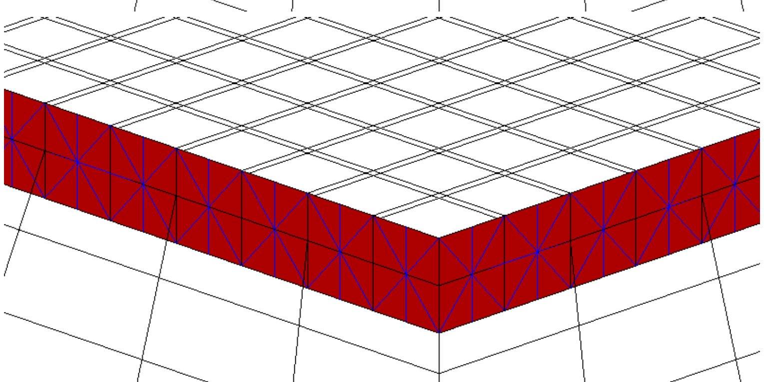
# Overview



Start out from a input file containing shell and solid elements which are not connected.



Using the element geometries, detect matching shell edges and solid faces and create a set of coupling regions.



For all coupling regions, create a common refined mesh.

# How to obtain the common mesh

- Preferred method:
  - X. Jiao, M. Heath, Common-refinement-based data transfer between non-matching meshes in multiphysics simulations, Int. J. Numer. Meth. Engng 2004; 61:2402–2427
  - Implemented in C++ in B2000++.
- Alternative method (prototype):
  - Implemented in Python, using high-precision numerics (mpmath module).
  - For each coupling region, create a parametric surface.
  - Project the shell and solid surface meshes to this parametric surface.
  - Peform a 2D constrained Delaunay triangulation on the parametric surface.
  - Project the resulting triangles back to the shell and solid surface meshes.

# Implementation issues

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- The meshing algorithm needs to be robust:
  - Even for the simplest cases, numerical problems occur due to limited numerical precision.
  - Points that are very close together produce triangles with very large angles. This may lead to an invalid mesh (e.g. inverted triangles or overlapping triangles).
  - The largest part of the implementation and testing effort is concerned with achieving this robustness.
- Testing of the meshing algorithms was performed on a large number of test cases (automated procedure).

# Formulation of the constraints

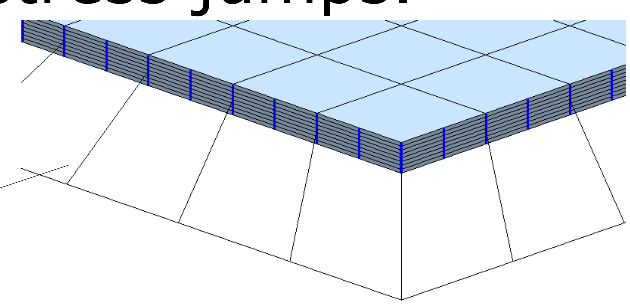
- At the beginning:
  - Determine the set of target shell d.o.f. which define the position of the shell surface. For each of these *m* d.o.f.'s, there will be a constraint.
  - For each triangle, determine the integration order (depends on the shell and solid elements).
- For each integration point in each triangle:
  - Evaluate displacement of the point on the solid surface ($u_1$) and on shell surface ($u_2$), their derivatives of $u_2$, and the area ($A$) associated to the integration point.
  - For each target shell d.o.f. $u_i$ ($1 <= i <= m$), add to the constraint $q_i$ the value of $A$ * $d\_|u_1-u_2|/d\_u_{2,i}$.

# Formulation of the constraints

- First and second variation:
  - These constraints are evaluated for the first variation.
  - The derivatives of the constraints w.r.t. all d.o.f.'s are needed for the second variation. This is a sparse ($n$ x $m$) matrix, with $n$ being the total number of d.o.f.'s.
- Evaluation of values and derivatives:
  - Solid elements: Derivatives are constant (shape functions). The constraints are linear.
  - Shell elements: Values depend on the rotation of the shell directors, hence, the constraints are nonlinear and the derivatives are non-constant.
  - In B2000++, the constraints and derivatives thereof are evaluated by means of the solid and shell element implementations.

# Additional necessary constraints

- Numerical tests have shown that for certain geometries and material definitions, the common refined mesh L2-method under-constrains the solid part of the interface.
- This may result in severe stress jumps near the interface.
- To amend this, we implemented additional point-wise contraints to ensure that the solid surface remains flat in the transverse direction. These constraints are independent of the shell region and the $L_2$-minimization.
- This eliminates the observed stress jumps.



# How to use

- In the input file, the user must specify the 'add_field_transfer_coupling' directive.
- When this directive is present, the pre-processor automatically creates common meshes between matching shell and solid interfaces.

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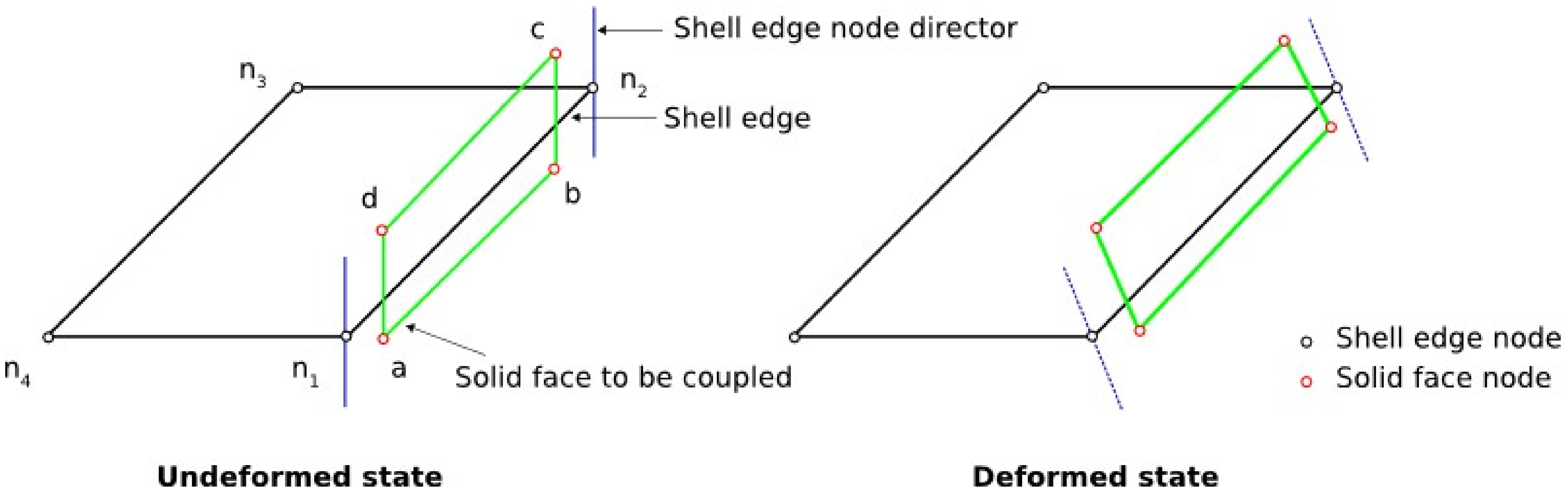
# Solid-to-shell coupling elements (SSC)

- A point-wise method. The shell region (coarse mesh) is the source region, the solid region (fine mesh) is the target region.
- The solid nodes are constrained to the shell surface.



# Solid-to-shell coupling elements

Features:

- 'TF' elements (transverse-free). Eliminates artificial stress concentrations due to shell/solid formulation differences.
- Automatic insertion using the '`add_ssc_elements`' directive, integrated in B2000++ pre-processor.

Remaining problems: Stress concentrations due to point-wise coupling.

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# Test case definition

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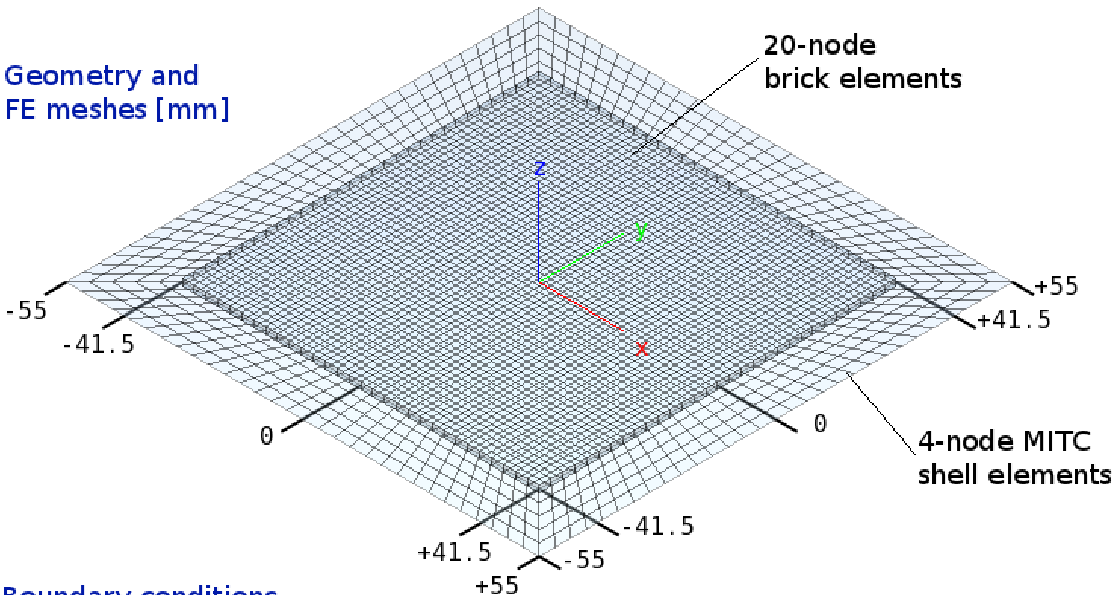
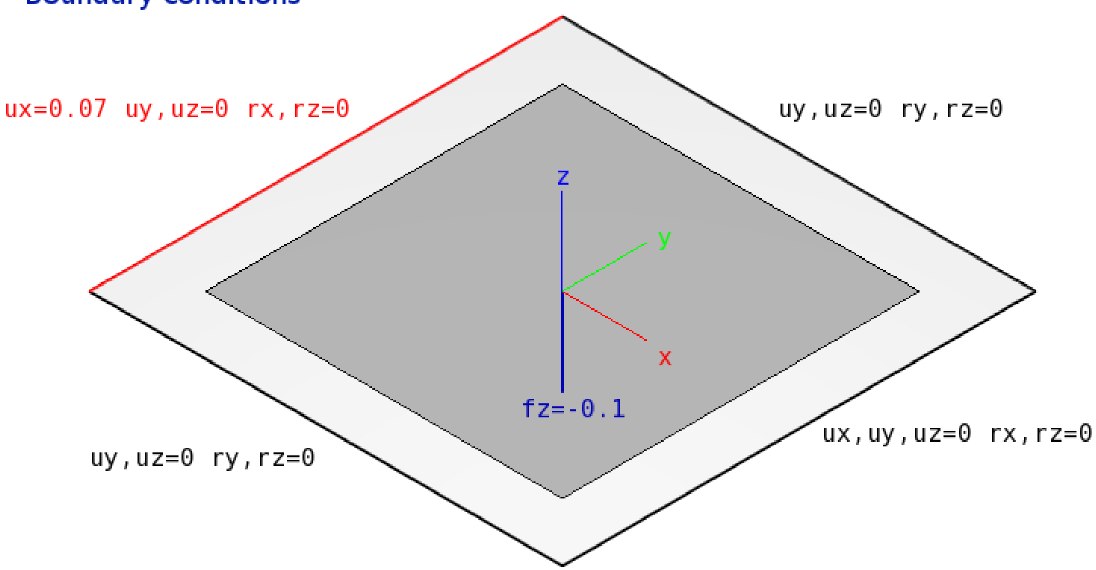
*Provided by CIRA*

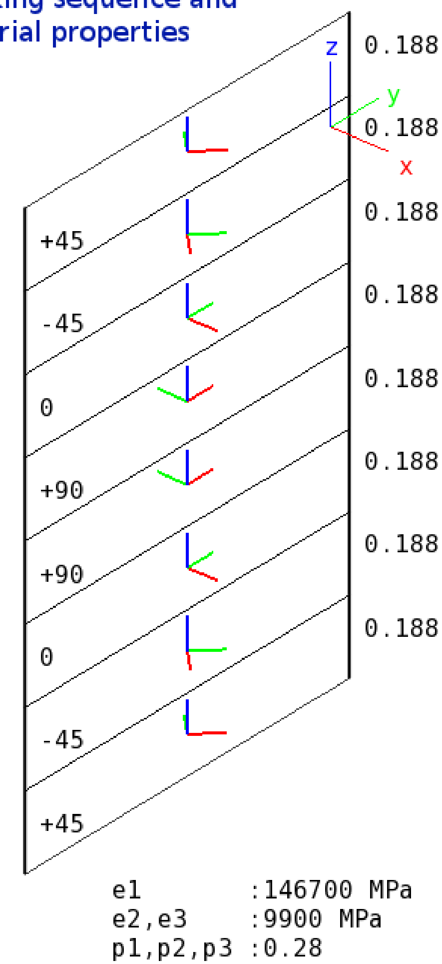
Geometry and FE meshes [mm]




Boundary conditions



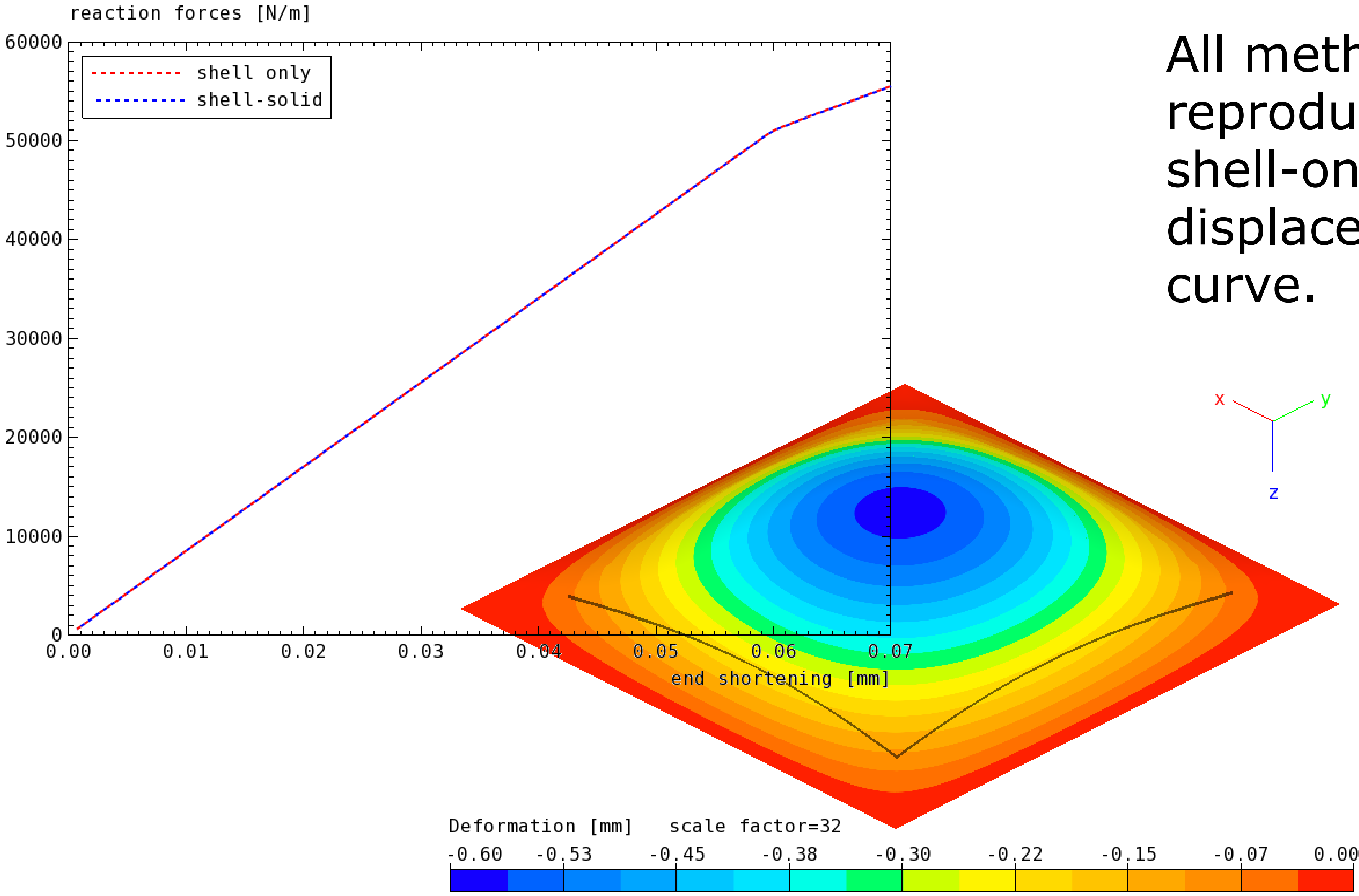

Stacking sequence and Material properties



# Test case goals

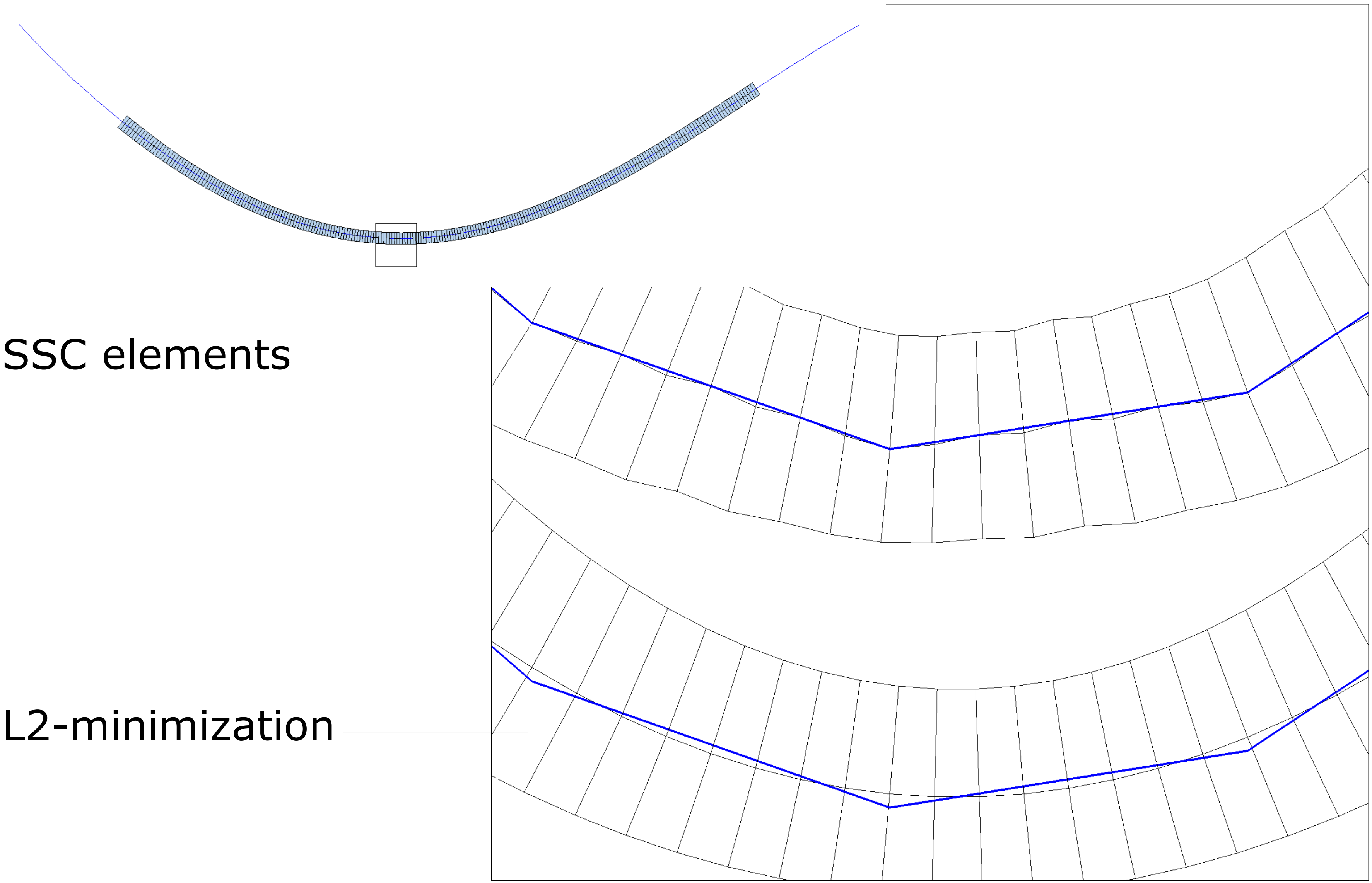
- The shell-solid kinematic coupling should:
  - Maintain load-displacement curve (conservative),
  - Remain free of stress jumps inside the solid region (no overconstraining),
  - Have little effect on the convergence of the Newton iterations.
- All this should be maintained even when the mesh density of the solid region is much higher than that of the shell region.
- We will evaluate:
  - Load-displacement curves,
  - Sxx and Szz though the centre and along the interface,
  - Results of the $L_2$-minimization method and the solid-to-shell coupling elements (SSC).

# Load-displacement curves and deformation



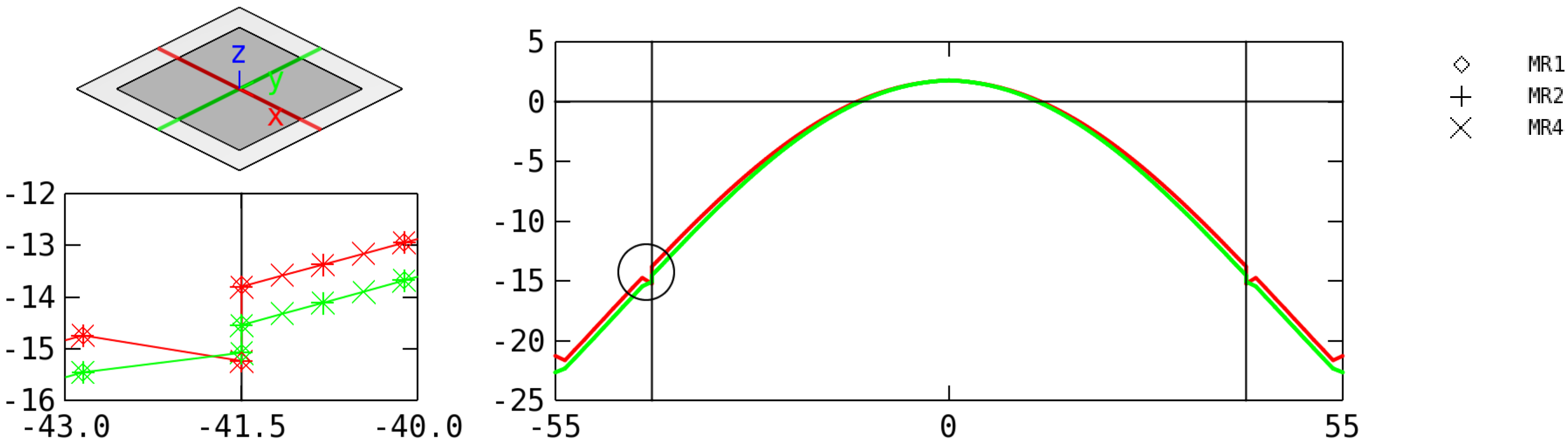

All methods reproduce the shell-only load-displacement curve.

# Deformation along shell-solid interface

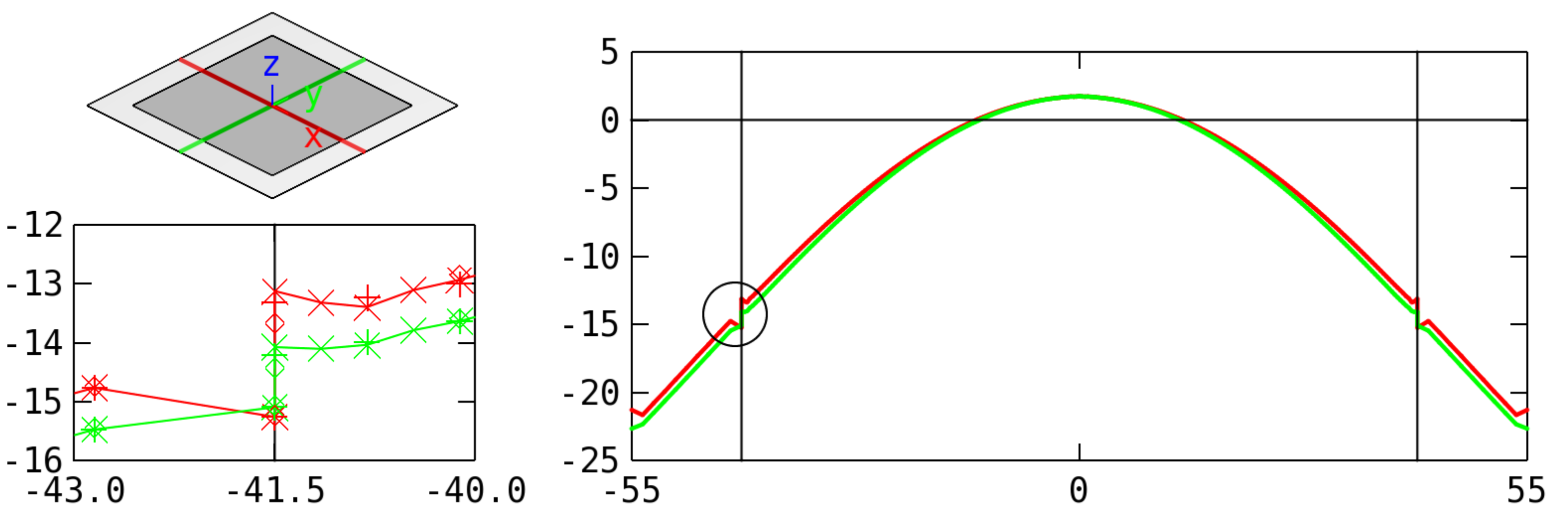


# $\sigma_{xx}$ through the centre

Sxx [MPa] at lower surface, L2-minimization




Sxx [MPa] at lower surface, SSC elements



# $\sigma_{xx}$ along the shell-to-solid interface

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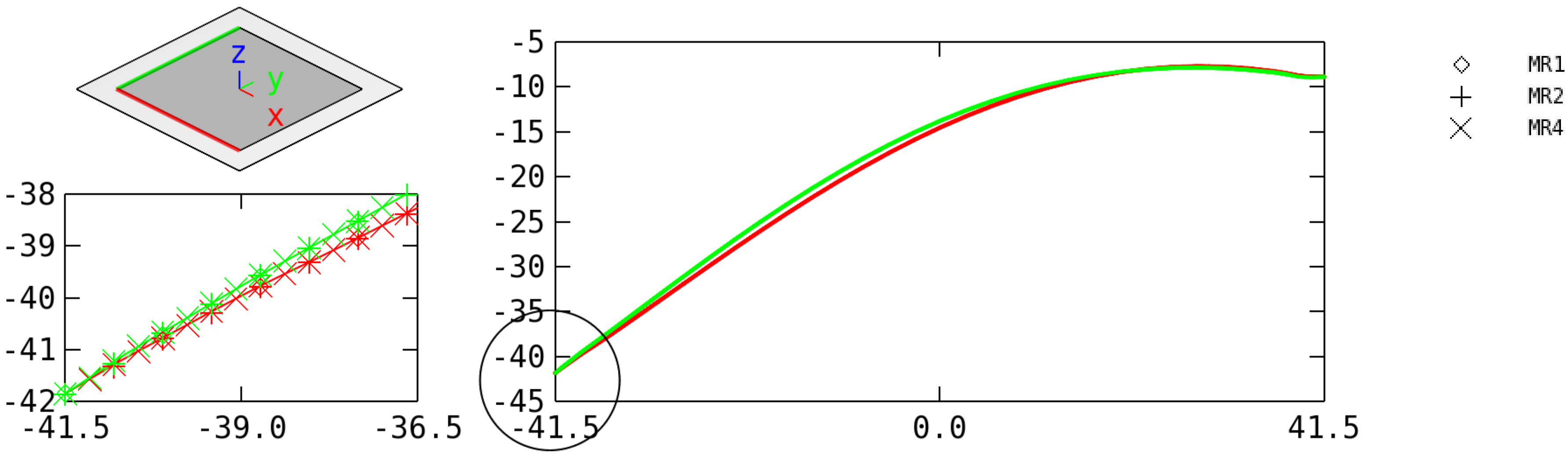
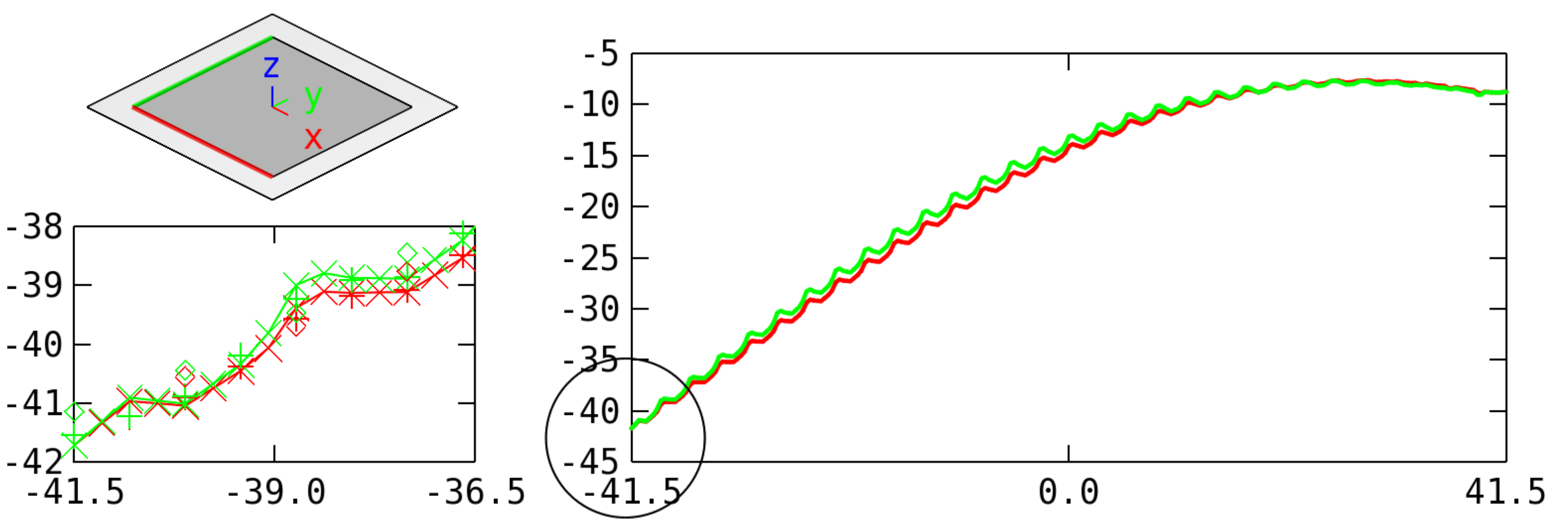
Sxx [MPa] at lower surface, L2-minimization

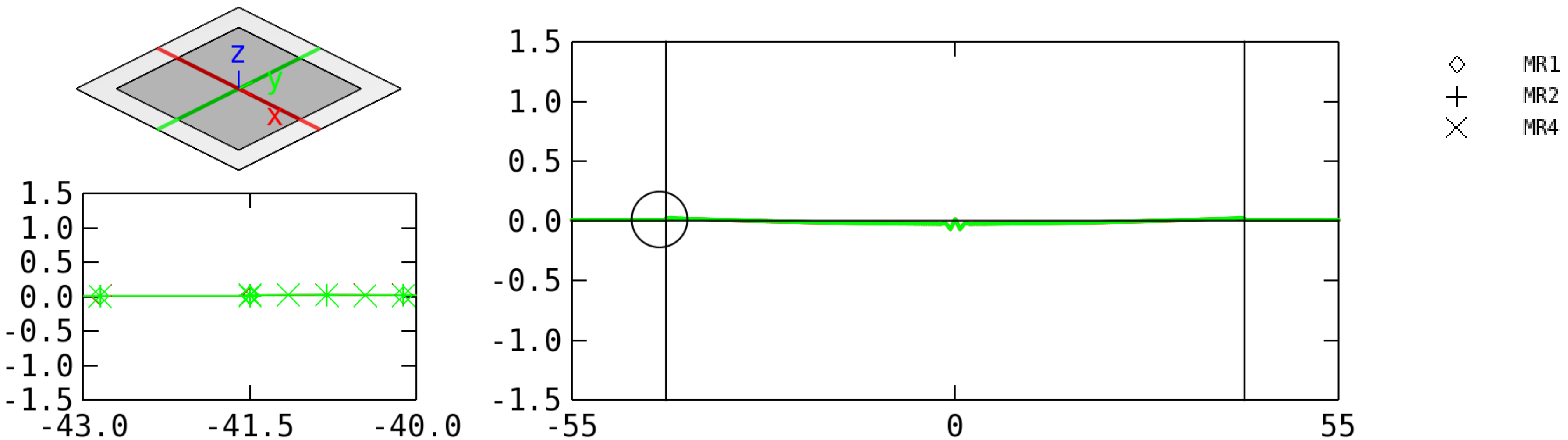



Sxx [MPa] at lower surface, SSC elements

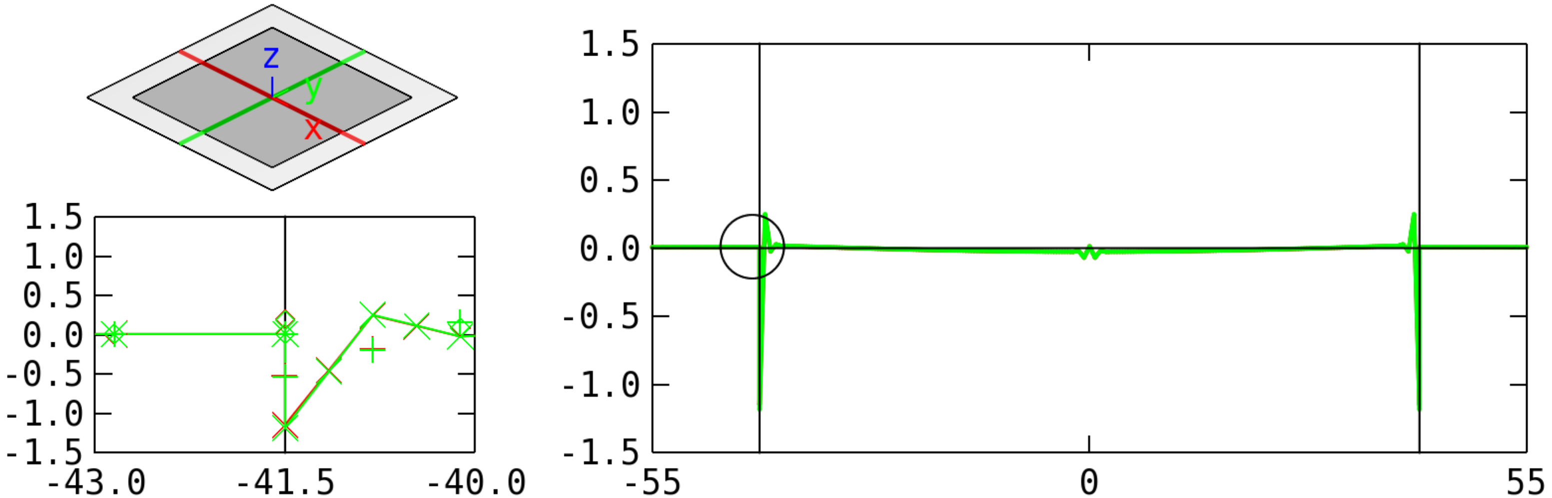


# $\sigma_{zz}$ through the centre

Szz [MPa] at lower surface, L2-minimization

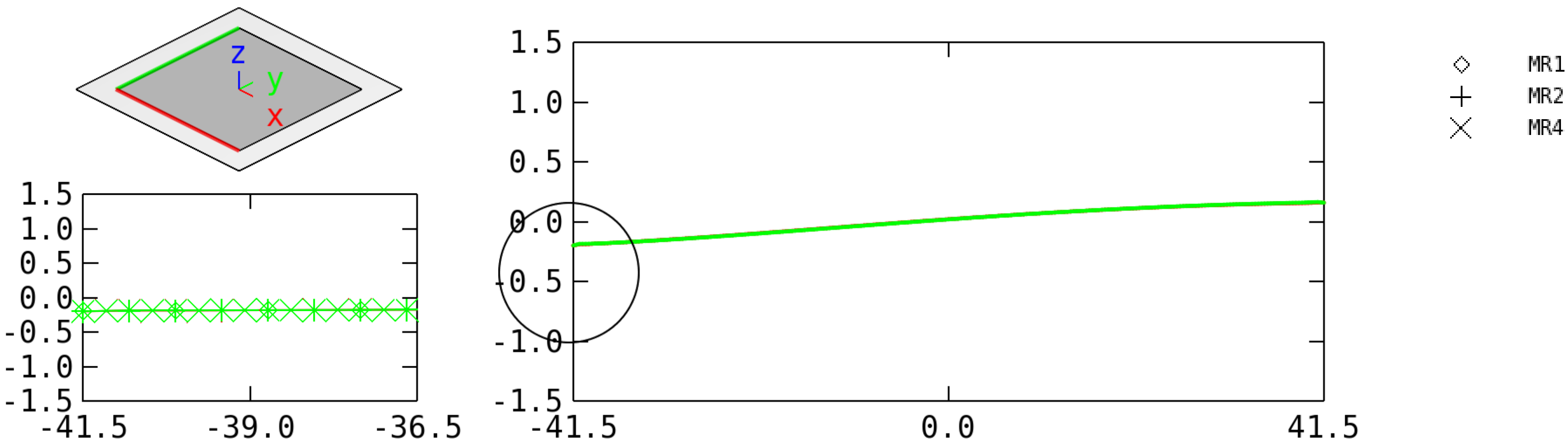



Szz [MPa] at lower surface, SSC elements

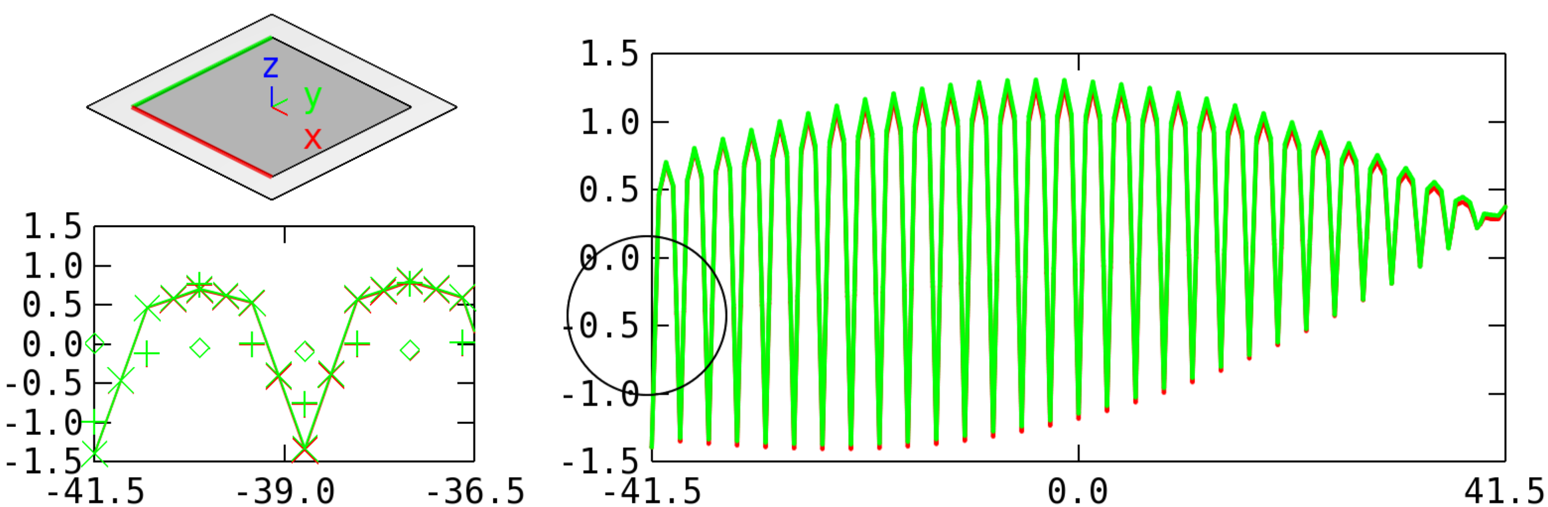


# $\sigma_{zz}$ along the shell-to-solid interface

Szz [MPa] at lower surface, L2-minimization




Szz [MPa] at lower surface, SSC elements



# Test case results summary

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- Both methods converge, but the convergence for the $L_2$-minimization method is better.
- Both methods accurately reproduce the load-displacement curve.
- The $L_2$-minimisation method produces a smooth stress distribution in the whole solid region.
- The solid-to-shell elements exhibit stress jumps near the interface; this is due to overconstraining.

***Thank you for your attention!***